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Building an understanding of sketching function derivative graphs through the APOS approach

Enny Listiawati[1*], Dwi Juniati[2], Rooselyna Ekawati[3]
[1,2,3]Mathematics Education, Faculty of Mathematics and Natural Science, Universitas Negeri Surabaya, Surabaya, Indonesia; ennylistiawati@stkippgri-bkl.ac.id (E.L.) dwijuniati@unesa.ac.id (D.J.) rooselynaekawati@unesa.ac.id (R.E.)
[1]Department of Mathematics Education, STKIP PGRI Bangkalan, Bangkalan, Indonesia.

Abstract: The derivative of a function is a basic concept in calculus with various applications in the STEM fields. A deep understanding of this concept is essential for students to face the challenges of analysis, optimization, and application in science and technology inquiries. However, the understanding of derivatives of functions among the population of prospective mathematics teachers in Indonesia is still low, specifically when it comes to sketching graphs of derivatives of functions and the application of the concepts. The APOS (Action, Process, Object, Schema) theory approach seems to hold a relevant solution needed to improve this understanding. Previous studies have shown that most students still have difficulty understanding the relationship between graphs and derivatives of functions. In the present study, APOS theory was used to analyze the subjects' understanding of the concept of derivatives, especially related to sketching graphs of derivatives of functions by describing their mental structure and mechanisms completely. The results of the study revealed that the subjects' understanding had reached the Schema stage. However, there was no mental interiorization mechanism since the subjects had difficulty at the Process stage. Consequently, the subjects could not explain the steps to draw a graph of derivatives of functions from the given function graph. Finally, this study highlights the importance of more effective teaching strategies to strengthen students' conceptual understanding of derivatives of functions, especially sketching derivative graphs.
Keywords: *APOS, Derivatives, Graphics, Mathematics.*

1. Introduction

The derivative of a function is a fundamental calculus concept that is important in the STEM disciplines [1, 2]. The concept is used as a step in function analysis, optimization, understanding changes in phenomena, and practical applications in sciences, economics, and higher education [3, 4]. However, students often struggle to understand the concept of the derivative of a function [5] especially in chain rules, graphs, and derivative applications such as optimization and critical point velocity calculations [6].

Responding to students' problems in understanding the stated concept, this study highlights the importance of the APOS (Action, Process, Object, Schema) method in improving the understanding of function derivatives, especially in the concept of sketching function derivative graphs. In APOS theory, the mental structure of Action, Process, Object, and Schema is an individual's construction when learning certain mathematical concepts [7]. The mechanisms for building mental structures (mental mechanisms) that can be carried out by an individual include interiorization, encapsulation, coordination, reversal, de-encapsulation, and thematization [8]. The theory further explains that learning mathematical concepts involves applying transformations to already constructed objects and constructing new mental or cognitive objects through these transformations (Moll, et al, 2016). Thus, it

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* Correspondence: ennylistiawati@stkippgri-bkl.ac.id

considers understanding mathematical concepts as a gradual process of building and explaining relationships between cognitive structures [5].

The APOS theory has been used in previous studies. One example [9] studied prospective mathematics teachers' understanding on integral based on APOS theory with regard to gender differences. In the study, the data were collected through assignments on integral and interviews. The results of data analysis showed that at the actions stage, subjects solved the integral problem procedurally but at the schemes stage, the coherence of the two subjects'schemes was different. On the other hand, another study Maf'ulah and Juniati [10] reports that by using the M-APOS approach method, only 28% of the subjects were able to define function graphs, while 62% of them were able to describe functions but showed difficulty in understanding the two-way relationship between functions and graphs. The APOS theory was also utilized to find out students' understanding when sketching quadratic function graphs, and the study shows that there were differences in understanding based on gender; the male subjects directly sketched the graphs, while the female subjects were more analytical in using tables and coordinates [11]. Furthermore, it was reported that the understanding of the graph of inequality of two variables can be expanded through the concept of action, process, and variation to solve optimization problems more effectively [12]. Then, the use of APOS shows that most prospective teachers were still at the action level in understanding the concept of quadratic functions, with procedural understanding in one study Mutambara, et al. [13]. The APOS theory has also been used by another study Fuentealba, et al. [14] to identify and characterize the development of students' understanding of derivatives through cluster analysis and cognitive levels. Another research report [15] identifying students' errors in solving cases of critical thinking skills from two variables of linear equation based on the Action-Process-Object-Schema (APOS) theory. The findings showed that the students have low critical thinking skills; therefore, they could not complete the task correctly and they could not reach a correct Schema.

In addition, a recent inquiru [16] used APOS theory APOS-ACE (Action, Process, Object, and Schema-Activities, Classroom discussion, and Exercises) to explore the teaching and learning of derivative by giving emphasis on its graphical understanding. For this purpose, a Genetic Decomposition is developed based on the outcomes of previous studies and on personal teaching experiences. Furthermore, one study [17] identify the epistemological obstacle of pre-service mathematics teachers on the basic concept of derivatives. These data are then reviewed by the framework of APOS theory (Action-Process-Object-Schema) to overview the thinking of processes experienced by participants in answering the questions given. The results of the study show that there is still some learning obstacles experienced by participants in understanding the basic concepts of derivatives. Another study Baye, et al. [18] employed mixed methods experimental (intervention) design within an APOS paradigm. The results of the qualitative data analysis revealed that the learning milieu created a positive impact on students' understanding of the concept of limit. Additionally, students provided coherent and viable reasons while making mental constructions and their coordination in the learning process based on the genetic decomposition grounded in APOS theory. Thus, the APOS theory is proven to hold importance in mathematics education as it is able to provide a deeper framework in comprehending how students build mathematical concepts gradually. Furthermore, it allows for better analysis of the development of students' understanding of mathematical concepts, by considering the cognitive processes involved, and provides more comprehensive insights into the effectiveness of teachings of mathematics and more objective measurement of learning outcomes. This study introduces an innovative approach using the APOS method involving the mental structures of Action, Process, Object, and Schema as well as the mental mechanisms of interiorization, encapsulation, coordination, de-encapsulation, reversal, and thematization to see students' understanding of function derivatives, especially in sketching function derivative graphs.

Theoretically, this study is expected to provide a deeper outlook on the mental structure and mental mechanisms involved in the concept of derivatives, especially sketching graphs of derivatives of functions. By using the APOS theory, this study hopes to enhance understanding of how students integrate actions, processes, objects, and schemas in understanding mathematical materials. Furthermore, this enriches knowledge about how students gradually build mathematical understanding and provides new insights into how these mental constructions play a role in mathematics learning. Practically, the results of this study will provide guidance for educators to design more effective and interactive teaching methods, by considering the steps that students need to master at each stage of APOS. This study also provides a basis for developing more appropriate evaluation tools to measure students' understanding of the concept of derivatives, and can be applied in a broader context of mathematics learning. Thus, the results of this study are expected to support the improvement of the quality of mathematics teaching and help create a more adaptive learning experience for students especially in Indonesia.

2. Research Methodology

2.1. Design

This study aimed to describe the understanding of prospective mathematics teachers in sketching function derivative graphs based on APOS theory, by considering the level of mathematics anxiety [19, 20]. It particularly focused on two groups of students with low and high mathematics anxiety. Through function derivative tasks and interviews, students' understanding was explored in the context of sketching function derivative graphs [11, 21, 22]. Furthermore, the APOS theory was utilized to test students' mental structures, including Action, Process, Object, and Schema as well as their mental mechanisms comprising interiorization, encapsulation, coordination, de-encapsulation, reversal, and thematization. Additionally, mathematics anxiety was measured through aspects of learning, class participation, and exams. The data obtained, in the form of oral and written descriptions, were analyzed exploratively in a qualitative manner, emphasizing students' actual understanding of the concept of derivative functions.

2.2. Research Subject

This study involved 26 prospective mathematics teacher students with high and low levels of mathematics anxiety consisting of 8 males and 18 females. The category of mathematics anxiety levels based on the score division was considered low if the score was in the interval of 15 to 37, while high mathematics anxiety was within the range of 38 to 60 [23]. From the mathematics anxiety questionnaire [24] given to the subjects, there were 4 male students in each category of low and high mathematics anxiety levels. Meanwhile, of the 18 female students, there were 6 students with low mathematics students and 12 subjects with high anxiety. This study explored qualitatively how mathematics anxiety affects students' understanding of the concept of derivatives. Moreover, this study took a female subject with low math anxiety as the sole research subject.

2.3. Procedures and Data Collection

This study utilized function derivative tasks and in-depth interviews as research instruments with the researchers as data collectors and analysts. The supporting instruments included a math anxiety questionnaire, a math ability test, and interview guidelines [4, 25]. The mathematics anxiety questionnaire [26] and mathematics ability test were designed to assess the subjects' mathematics anxiety and mathematics ability, while the function derivative task based on APOS theory was used to determine students' understanding of function derivatives [27, 28]. All instruments had been validated by mathematics education experts prior to being disseminated to the research subjects. Data were collected through triangulation by repeating the function derivative task and interviews to ensure consistency of understanding. Data were also analyzed through data reduction, data presentation, and conclusion drawing, then validated by the research supervisors to evaluate the understanding of function derivatives [29, 30].

2.4. Operational Variables

The operational variables of mental structure consist of several indicators: Action refers to students' ability to draw function derivative graphs. Process involves explaining the steps in drawing function derivative graphs. Objects include determining the relationship between function derivative graphs. Lastly, Schema describes the ability to explain the nature of derivatives by connecting relevant actions, processes, and objects.

Table 1.
Operational variables.

Mental structure	Indicators	Code
Action	Sketching a graph of the derivative of a function	A
Process	Explaining the steps for sketching a function derivative graph	P
Objects	Determining the derivative relationship of two functions based on the given graph.	O
Scheme	Describing the properties of derived functions by relating actions, processes, objects, and other possible schemes to be used in completing derived tasks.	S

2.5. Research Procedures

This study analyzed four types of data obtained from the selected subject: the results of the mathematics anxiety questionnaire, the mathematics ability test, written assignments, and task-based interview transcripts [12]. The analysis used the APOS theory which includes four stages: Action (concrete action), Process (organizing actions), Object (understanding mathematical concepts as permanent entities), and Schema (mental structures that organize knowledge) [31]. In addition, the mental mechanisms based on APOS consisting of interiorization, encapsulation, de-encapsulation, coordination, reversal, and thematization were also analyzed. The analysis process followed the stages of Miles and HubermanIlyas, et al. [32]; Miles, et al. [33] and Miles and Huberman [34]: data reduction, data presentation, and conclusion. Relevant data were selected, simplified, and grouped to facilitate further analysis. For validity, triangulation, member checks, and proper references were used.

3. Results and Discussion

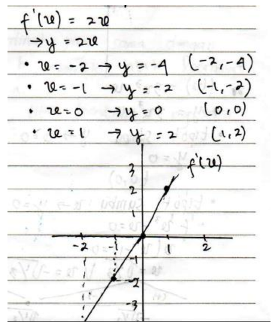
3.1. Results

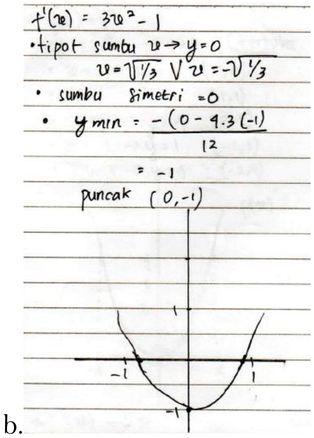
3.1.1. Action

The APOS method at the Action stage was applied by the subject to understand the concept of function derivatives, especially in sketching the function derivative graphs using the Function Derivative Task (FDT). At this stage, the subject was given tasks related to the material on sketching function derivative graphs at the Action stage using FDT as follows. The following function graph image $'$:

a. $f(x) = x^2 + 3$
b. $f(x) = x^3 - x - 4$

The subject was given an FDT to explore their understanding at the Action stage. Figure 1 presents data from the results of the subjects' written answers at the Action stage.

a. 

b. 

$f'(x) = 3x^2 - 1$

intersection point $x \rightarrow$
$y = 0$

$x = \sqrt{1/3}$ *or* x

Figure 1.
Subject work results action stage.

The subject sketched the graphs of derivative functions $f(x) = x^2 + 3$. She began by determining the derivative of the function, namely $f'(x) = 2x$. Next, the subject assumed $y = 2x$ and looked for some points (x, y) which corresponded with the equation $y = 2x$. From several points obtained, the subject sketched a graph by connecting the points. Next, the subject sketched a graph of the derivative of the function $f(x) = x^3 - x - 4$ by first determining the derivative of the function, namely $f'(x) = 3x^2 - 1$. After that, the subject determined the point where the graph intersected the coordinate axis, namely the point where it intersects the coordinate axis x and the point of intersection with the axis y. After that, the subject looked for the equation of the axis of symmetry using the formula $x = -\frac{b}{2a}$ which obtained $x = 0$. Then, the subject looked for the minimum value using the formula $y = -\frac{D}{4a}$ which obtained $y = -1$. From the equation of the axis of symmetry and the minimum value of the function, the peak of the parabola was $(0, -1)$. From this information, the subject connected the points obtained, namely the intersection points with the axis. x, the point of intersection with the y-axis, and the vertex of the parabola to sketch the graph $'(x) = 3x^2 - 1$). Based on this, it can be stated that in order to sketch a function derivative graph, the subject needed to first determine the function derivative. If the function derivative was a linear function, then the subject drew a function derivative graph by finding several points that were in accordance with the equation and connecting the points obtained so that a function derivative graph was formed in the form of a straight line. Furthermore, if the function derivative was a quadratic function, then the subject determined the point of intersection of the graph with the coordinate axis, namely determining the point of intersection with the coordinate axis x, the point of intersection with the axis y, the equation of the axis of symmetry, the optimum value, the peak of the parabola, and then connecting the points obtained so that a function derivative graph was formed in the form of a parabola.

3.1.2. Process and Interiorization

To find out the subject's understanding of the derivative of a function, especially about sketching, at the subject's Process stage, the FDT was given, and the subject's answer can be seen in Figure 2 as follows.

For example, given a function graph f:

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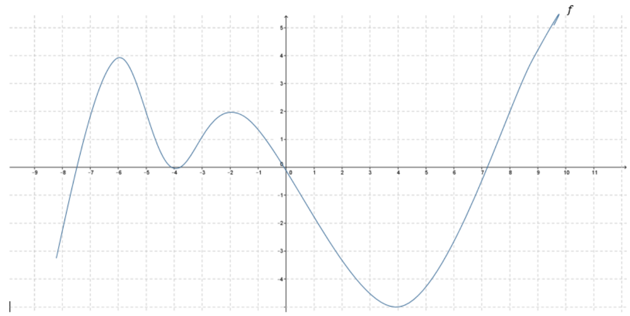


Figure 2.
Assignment of sketching a function derivative graph.

Explain how to draw a graph f' from the graph f!

The subject was given the FDT to explore her understanding at the Process stage, but the subject could not explain the steps to draw a function derivative graph from the given function graph so that it did not meet the Process indicator. Thus, there was no mental interiorization mechanism in explaining the steps to sketch a function derivative graph from the given function graph which was indicated by the subject being unable to explain the steps to sketch a function derivative graph from the given function graph.

3.1.3. Objects, De-Encapsulation, Coordination, and Reversal

To find out the subject's understanding of the sketching function derivative at the Object stage using FDT, the illustration was given in Figure 3.

The following image shows a graphic image of four functions indicated by the graph a, b, c, d

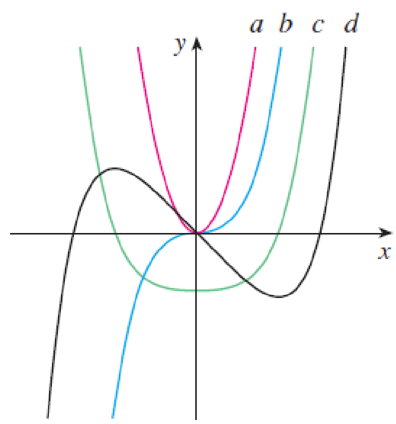
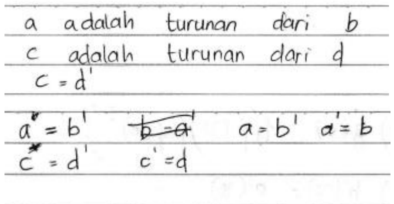


Figure 3.
Function graph.

Based on the graph, how is the relationship between the following functions?:

a. Function a and b?
b. Function c and d?
c. Derivative of function a and b?
d. Derivative of function c and d?

The subject was given FDT to explore her understanding at the Object stage. Figure 4 presents data on the results of the subjects' written answers at the Object stage.



a *is a derivative of* b

Figure 4.
The subject's answer.

After that, the subject determined the relationship between the derivatives of two functions based on the given graph, namely determining the relationship between the functions a and b and functions c and d based on the graph provided. This was done by looking at the similarity of the graph with her previous experience of working on questions so that a relationship of a derivative of b and c derivative of d was garnered. Based on this, it can be said that the subject determined the relationship between the derivatives of two functions based on the graphs given by using previous knowledge, namely by seeing the similarity of the graphs and functions from the previous questions so that the subject determined the relationship between the function and the parabola-shaped graph which was the derivative of the function with the graph having two peak points.

There was also a mental encapsulation mechanism in the form of transforming the process of sketching a function derivative graph into an object marked by the subject being able to determine the relationship of the derivatives of two functions based on the given graph, namely a function with a parabolic graph was a derivative of a function with a graph that had two peaks. The mental coordination mechanism occurred when the the subject combined two processes, namely the function graph, the function derivative graph, and the corresponding function. The mental reversal mechanism also occurred when the subject retraced the function derivative graph that corresponded to the original function by seeing the similarity of the function derivative graph and the function derivative in her previous knowledge. The mental De-Encapsulation mechanism occurred when the subject broke down the relationship between the derivatives of two functions into two mental structures. The process was determining the function graph of the given function and determining the function derivative graph of the given function.

3.1.4. Scheme

The subject was given FDT to explore her understanding at the Schema stage. She explained the nature of the derivative function by connecting actions, processes, objects, and other schemes that could be used to complete the derivative task of sketching a function derivative graph $f(x) = x^2 + 3$ that began by determining the derivative of $f(x) = x^2 + 3$ that was $f'(x) = 2x$. Furthermore, the subject assumed that $y = 2x$ and looked for some points (x, y) which satisfied the equation $y = 2x$. From several points obtained, the subject sketched a graph by connecting the points. Then, the subject sketched a graph of the derivative of the function $f(x) = x^3 - x - 4$ by first determining the derivative of the function, namely $f'(x) = 3x^2 - 1$. The subject determined the point where the graph intersected the coordinate axis. Next, the subject looked for the equation of the axis of symmetry $x = -\frac{b}{2a}$ which obtained $x = 0$. After that, the subject looked for the minimum value using the formula $y = -\frac{D}{4a}$ and obtained $y = -1$ and the peak of the parabola was $(0, -1)$. From this information, the subject connected the points obtained and sketched a graph $f'(x) = 3x^2 - 1$.

Based on this, it can be said that the subject explains the nature of the function derivative by connecting actions, processes, objects, and other schemes that can be used to complete the task of

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drawing a function derivative graph using the function derivative rules and the steps for sketching a function graph, namely determining the point of intersection of the graph with the coordinate axis, determining extreme points and determining other points on the graph. The mental mechanism of thematization occurred when the subject identified the suitability of the function derivative obtained with the steps used to draw a function derivative graph.

The subject's understanding of the function derivative on the topic of drawing a function derivative graph based on the APOS theory was presented in its entirety described in the stages of the mental structure of Action, Process, Object, and Schema, and based on the mental mechanisms that build it, namely interiorization, encapsulation, coordination, reversal, de-encapsulation, and thematization, which can be seen in Table 2. The subject's understanding had reached the schema stage, but there was no mental mechanism of interiorization as she failed to explain the steps for sketching a function derivative graph from the given function graph. Consequently, the subject's understanding did not reach the Process stage.

Table 2.
APOS method understanding achievement.

Mental structure/Mental mechanism	Indicator Code	Achievement of understanding
Action	A	Sketching a function derivative graph by first determining the function derivative. If the function derivative was a linear function, then the subject looked for several points that satisfied the equation and then connected the points obtained so that a function derivative graph was formed in the form of a straight line. If the function derivative was a quadratic function, then the subject determined the point of intersection of the graph with the x-axis, the point of intersection with the y-axis, the equation of the axis of symmetry, the optimum value, the peak of the parabola, and connected the points obtained so that a function derivative graph was formed in the form of a parabola.
Interiorization		There was no mental mechanism of interiorization because the subject could not explain the steps for sketching a function derivative graph from a given function graph.
Process	P	The subject could not explain the steps for drawing a function derivative graph from a given function graph.
Encapsulation		The subject could determine the relationship between the derivatives of two functions based on the given graph, namely a function with a parabolic graph was a derivative of a function with a graph that has two peak points.
Objects	O	The subject determined the relationship between the derivatives of two functions based on the graphs given by using previous knowledge, namely by seeing the similarity of the graphs and functions from the previous questions so that the subject determined the relationship between the function and the parabola-shaped graph which was the derivative of the function with the graph having two peak points.
Coordination		The mental coordination mechanism occurred when the subject combined several processes, namely function graphs, function derivative graphs, and corresponding functions.
Reversal		The mental reversal mechanism occurred when the subject retraced the function derivative graph that corresponded to the original function by seeing the similarity between the function derivative graph and the function derivative in her previous knowledge.
De-Encapsulation		The mental mechanism of De-Encapsulation occurred when the subject broke down the relationship between the derivatives of two functions into two mental structures. The process was determining the function graph of a given function and determining the function derivative graph of a given function.

Scheme	S	The subject explained the nature of the derivative of a function by connecting actions, processes, objects and other schemes that can be used to complete the derivative task of drawing a graph of the derivative of a function using the rules of the derivative of a function and the steps for drawing a graph of a function, namely determining the point of intersection of the graph with the coordinate axis, determining extreme points and determining other points on the graph..
Thematization		The mental mechanism of thematization occurred when the subject identifies the correspondence of the obtained function derivative with the steps used to draw the function derivative graph.

4. Discussion

This study generated findings on understanding of function derivatives, especially in sketching function derivative graphs presented based on the APOS level of understanding. At the Action stage, the subject sketched the function derivative graphs in several stages and approaches. The first approach focused on determining function derivatives using appropriate derivative rules, while the second approach used function graph sketching rules that were adjusted to the function derivatives obtained. This finding is in line with a previous study Trigueros, et al. [35] subjects perform complex analyses involved in deeply understanding the behavior of functions when drawing or understanding their graphs. Understanding how students use their knowledge in completing tasks provides information about strategies that can be used when teaching function derivatives to students. Seeing the interesting strategies used by students helps to understand the difficulties involved in these interactions. The importance of understanding the concept of function derivatives in sketching function derivative graphs can be seen from the levels of the APOS mental structure of Action, Process, Object, and Schema which is complemented by analyzing mental mechanisms coherently. This can provide complete information about understanding the concept of function derivatives. In addition, students' abstraction abilities in learning mathematics are greatly influenced by the cognitive stages in the APOS theory (Action, Process, Object, Schema). This finding reinforces the view that understanding mathematical concepts involves the relationship between actions, processes, objects, and schemes at various levels of cognitive development. In addition, these findings are expected to help overcome students' difficulties in working on derivative problems and can improve student achievement. This is based on previous research Nga, et al. [36] which shows that students who participate in learning activities based on APOS theory experience increased academic achievement and attitudes. In addition, APOS theory improves students' ability to find solutions to derivative problems.

The study is also in line with one research study Fuentealba, et al. [14] which shows that students are able to explain number sequence patterns by connecting actions, processes, and objects in mathematics learning. This understanding illustrates the importance of the cognitive modeling process in generalizing patterns and connecting them to other concepts. This approach is also relevant in the context of learning derivations, where sketching a function derivative graph requires strong visualization skills, both for linear and quadratic functions. However, the current research's findings show that the subject had difficulty explaining how to sketch a function derivative graph from a given function graph so that there was no mental interiorization mechanism that connected the APOS Action to Process stages.

This study also reinforces the findings of a research paper [14] regarding the application of the APOS method in understanding double integrals, which shows that although many students have difficulty, most succeed in achieving a deeper understanding of the concept. In the process of sketching a derivative graph as shown in the present study, the subject showed recognition of the form of the derivative function, both linear and quadratic, explained the steps needed to find the intersection and vertex of a parabola, and determined the relationship of the derivative of two functions based on the given graph. This understanding shows the importance of the concept of the derivative of a function not only for problem-solving, but also for sketching and analyzing the derivative graph of a function more effectively. By following clear steps, the subject emphasizes that a deep understanding of derivatives is very useful in various mathematical contexts.

5. Conclusion and Suggestions

This study has successfully described the systematic process in drawing function derivative graphs with various approaches, involving the use of function derivative formulas and the application of graph drawing rules based on the given function. This approach not only deepens the understanding of function derivatives but also connects it with visual understanding which is important in sketching function derivative graphs. This finding is also in line with the APOS theory, which emphasizes the importance of the relationship between the mental structure of Action, Process, Object, and Schema and the mental mechanisms that build them, namely interiorization, encapsulation, coordination, reversal, de-encapsulation, and thematization in understanding the concept of derivatives. The findings also show that the subject's understanding reached the APOS Scheme stage, but the subject had difficulty at the Process stage. Furthermore, the application of the APOS theory can help deepen students' cognitive processes in understanding the concept of derivatives through various stages, starting from Action, Process, Object, to Schema. In addition, visual and analytical understanding of students in sketching derivative graphs must be strengthened, especially in the context of learning. As for research purposes, future studies need to pay attention to differences in students' levels of mathematical anxiety, as well as provide a deeper approach to understanding mathematical concepts.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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