

Assessing the efficiency of Pacific Asia Islamic stock markets using multifractal detrended fluctuation analysis

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Abstract: This paper aims to assess the informational efficiency of six Islamic stock markets in the Asia-Pacific region. To achieve this, the study pursues two specific objectives: first, to capture and quantify the degree of multifractality, and second, to identify the sources of this multifractality within these markets. Utilizing the Multifractal Detrended Fluctuation Analysis (MF-DFA) method, the multifractal behavior of these markets is analyzed. The empirical findings, derived from the application of generalized Hurst exponents, Rényi exponents, and the singularity spectrum, indicate that the six Islamic markets studied display multifractal properties, suggesting a lack of market efficiency. Furthermore, by employing shuffling and phase randomization techniques, it is determined that long-term correlations and heavy-tailed distributions jointly contribute to the observed multifractality. The results of this study have several practical implications for various market stakeholders. Investors should incorporate multifractal analysis into their strategies, focusing on long-term correlations and extreme events to enhance predictive accuracy and risk management. Diversification can help mitigate risk, while remaining vigilant about long-term trends and market patterns. Policymakers should improve market transparency, address inefficiencies, and implement targeted regulations to enhance market stability, such as stress-testing for extreme events and long-term trends. Future research should explore the factors driving multifractality to improve financial models.

Keywords: Efficiency, Generalized Hurst exponents, Multifractality, Rényi exponents, Spectrum.

JEL Classification: C12; C13; G14; G15.

1. Introduction

The efficiency of stock markets is a subject that has captivated the attention of economists, investors, and researchers for several decades. The Efficient Market Hypothesis (EMH), initiated by Fama [1], suggests that the prices of financial assets fully reflect all available information. If this hypothesis is valid, it becomes impossible to predict price movements or achieve systematic profits through the analysis of historical data. However, despite its popularity, the EMH has been widely contested, notably due to the inherent complexity of stock market operations and observed anomalies.

To evaluate the weak form efficiency hypothesis, various statistical tests, both parametric and non-parametric, have been used [2]-[9]. These tests include the unit root tests, the normality tests, the Ljung-Box serial correlation test, the variance ratio test, and the Runs test. The objective of these tests is to verify the Random Walk Hypothesis (RWH). All the statistical tools commonly used in the literature to test the RWH share a significant limitation: they fail to account for the long memory, non-linearity and fractal dimensions at different scales often inherent in financial time series.

Studying weak form informational efficiency is vital for financial markets as it enhances resource allocation, informs investment strategies, improves risk assessment, reduces information asymmetry, and supports market stability. It also drives financial innovation and aids in fraud detection, ensuring a fair and transparent market environment.

In the context of emerging Islamic markets, such as those in the Asia-Pacific region, studying informational efficiency is particularly important. These markets are often characterized by greater volatility, less developed structures, and lower transparency compared to mature markets. These characteristics can influence how information is incorporated into asset prices, which in turn can affect overall market efficiency. These challenges underscore that these financial markets are not linear and predictable systems but rather complex ones. Their level of complexity varies across different time scales, characterized by long-term correlations and heavy-tailed distributions, both of which contribute to the multifractal behavior.

In this paper, we will adopt an innovative approach that has gained popularity in recent years, the Multifractal Detrended Fluctuation Analysis (MF-DFA) method (Kantelhardt et al., [10]). This technique allows for the exploration of multifractal properties of financial time series, revealing variations at different scales and uncovering complex behaviors that might be overlooked by traditional analysis methods. The MF-DFA method is an extension of the Detrended Fluctuation Analysis (DFA) (Peng et al., [11]). The DFA method was designed to overcome the limitations of traditional time series analyses, which were often hindered by the presence of trends or non-stationarities, making it difficult to detect long-term correlations. DFA allows for the removal of these local trends and measures the residual fluctuations at different scales, making it a valuable tool for analyzing complex data. While DFA focuses on identifying long-term correlations, MF-DFA enables the examination of behaviors across multiple scales, capturing the multifractal nature of certain time series.

The primary goal of this paper is to evaluate the informational efficiency of six Islamic stock markets of the Asia-Pacific region. To accomplish this, the study focuses on two key objectives: first, to capture and measure the extent of multifractality in these markets, and second, to determine the underlying sources contributing to this multifractality. The analysis is conducted using the MF-DFA method to examine the multifractal behavior of these markets. We limited our study to six Islamic stock markets from Asia-Pacific region for which data has been available since January 1, 2011.

There is a research gap in the investigation of multifractal behavior in Asia-Pacific Islamic stock markets. This research aims to fill this gap by advancing knowledge, offering new perspectives, and providing practical implications for investment strategies, risk management, and regulatory approaches. Additionally, it will contribute to the academic field of Islamic finance and offer insights into market stability and volatility.

This article is structured as follows: Section 2 provides a literature review on studies investigating multifractality in stock market exchanges. Section 3 outlines the data and methodology used. The empirical results and discussion are presented in Section 4, followed by the conclusion in Section 5.

2. Literature Review

This literature review explores the growing application of Multifractal Detrended Fluctuation Analysis (MF-DFA) in financial markets, particularly in capturing multifractality in stock market exchanges. Several studies have highlighted the importance of multifractal analysis in understanding market behavior, revealing its potential in identifying inefficiencies and complexities that are not captured by traditional financial models.

For example, Norouzzadeh and Rahmani [12] applied MF-DFA to analyze the Iranian rial to US dollar exchange rate, uncovering multifractal properties such as scaling and generalized Hurst exponents. Their findings pointed to fat-tailed distributions and nonlinear temporal correlations as key contributors to the multifractality observed in the exchange rate. Similarly, Xinsheng et al [13] extended this approach to the Chinese stock index futures market, showing that returns exhibit both long-term correlations and multifractality. They found that long-term correlations were the predominant source of multifractality, while fat-tailed distributions also played a role.

Further studies, like Caraianni [14], focused on the multifractality of daily returns from various European stock indices, including the Czech PX, Hungarian BUX, and Polish WIG indices. His work found that the global Hurst exponent varied across different moments, suggesting a multifractal spectrum. Shuffling the data did not alter its multifractal properties, reinforcing the robustness of multifractality in these markets. Additionally, the author examined the impact of the 2008-2009

financial crisis on these indices and observed complex shifts in the multifractal spectrum, though he found no clear evidence of increased multifractality during the crisis. This indicates that while crises may impact market behavior, the underlying multifractal nature of stock returns remains resilient.

El Alaoui and Benbachir [15] studied the Czech, Hungarian, and Russian stock exchanges and used permutation and surrogation techniques to confirm that these markets exhibited multifractality and inefficiencies. This aligns with the findings of Wang et al. [16], who examined the NASDAQ Composite Index and identified leptokurtic returns and long-term memory as primary sources of multifractality. Wang et al.'s study [16] highlighted that multifractality in stock returns is a feature of markets that fail to achieve weak-form efficiency.

Rui et al. [17] expanded the scope of MF-DFA to major stock indices from Europe, America, and Asia, covering the period from 2006 to 2013. Their findings confirmed the presence of multifractal properties in all indices, with the Hang Seng Index, Dow Jones Index, and FTSE 100 Index exhibiting the highest risk in their respective regions. This demonstrates that multifractality is a global phenomenon, not limited to specific markets or regions. Similarly, Hasan and Mohammad [18] observed significant non-linearity and shifts in the singularity spectra during the 2007-2008 crisis in markets such as the U.S., Japan, Hong Kong, South Korea, and Indonesia, further reinforcing the idea that multifractality is influenced by market conditions, including periods of financial turmoil.

Studies by Tiwari et al. [19] and Hong-Yong and Tong-Tong [20] also contributed to the understanding of multifractality in sectoral indices and stock market exchanges. Tiwari et al. [19] analyzed the efficiency of Dow Jones sector ETFs and identified varying degrees of multifractality across sectors, with utilities and consumer goods ETFs being more efficient than financial and telecommunications ETFs. They also found that efficiency levels were significantly impacted by the 2008 financial crisis. Hong-Yong and Tong-Tong [20] explored the Shanghai Composite Index, Shanghai Bond Index, and Shanghai Fund Index, finding that multifractality was present across all three, with varying fractal characteristics in cross-correlations. These studies highlight that multifractality can differ across sectors and regions, further emphasizing the complexity of financial markets.

More recent research has continued to explore the multifractal properties of various financial markets. For instance, Zhu and Zhang [21] used MF-DFA to study the CSI 800 Index, finding that the multifractal properties of the logarithmic returns varied by weighting order, with more pronounced multifractality for certain indices. Gu and Huang [22] extended the analysis to high-frequency data from the Shenzhen Stock Exchange, confirming that fluctuations exhibited multifractality due to both long-term correlations and fat-tailed distributions. Similarly, Chenyu et al. [23] applied MF-DFA to foreign exchange markets, revealing significant multifractal properties in the EUR, GBP, CAD, and JPY exchange rates, with long-term correlations and fat-tailed distributions as the key contributors.

Faheem et al. [24] analyzed long-term dependence and multifractal parameters in stock indices from nine MSCI emerging Asian economies and found varying degrees of multifractality. This found that markets with higher long-term dependence, such as India and Malaysia, exhibited stronger multifractal behavior, aligning with the fractal market hypothesis. Datta [25] analyzed the exchange rates of USD, GBP, EUR, and JPY against the Indian rupee, identifying multifractal characteristics attributed to fat-tailed distributions in the case of USD and JPY, and both long-term correlations and fat-tailed distributions for GBP and EUR. Finally, Raza et al. [26] explored the efficiency of conventional and Islamic sectoral stock markets, finding that both markets exhibited multifractality and long-term memory, with differences observed during the COVID-19 pandemic in sectoral efficiency.

Overall, this body of literature underscores the widespread presence of multifractality in global financial markets, which arises from long-term correlations and fat-tailed distributions. The findings indicate that multifractal analysis provides valuable insights into market inefficiencies and complexities that traditional models may overlook.

In line with previous research, the current study aims to assess the banking efficiency of Islamic stock markets in the Asia-Pacific region using the MF-DFA method.

3. Methodology

3.1. Data

The data for this study consist of daily closing prices from Islamic indices across six stock markets in Asia-Pacific region: China, India, Indonesia, Pakistan, Malaysia and Thailand. These indices adhere to Islamic finance principles, prohibiting financial activities that do not comply with Sharia law. The indices include:

- The FTSE Shariah China which is designed to measure the performance of Shariah-compliant companies listed in China. It is a part of the FTSE Shariah Global Equity Index Series.
- The Nifty 500 Shariah Index is an Islamic equity index derived from the broader Nifty 500 Index, which tracks the performance of the top 500 companies listed on the National Stock Exchange of India (NSE) based on market capitalization.
- The Jakarta Islamic Index, which is a stock market index established on the Indonesia Stock Exchange (IDX).
- The Karachi Meezan 30 is a stock market index that tracks the performance of the top 30 Shariah-compliant companies listed on the Pakistan Stock Exchange (PSX).
- The FTSE Bursa Malaysia EMAS Shariah Index is a Shariah-compliant stock market index that comprises a subset of the FTSE Bursa Malaysia EMAS Index.
- The FTSE SET Shariah is designed to track the performance of Shariah-compliant companies listed on the Stock Exchange of Thailand (SET).

Throughout the remainder of the paper, we will refer to the six indices by China, India, Jakarta, Karachi, Malaysia and Thailand.

The data cover the period from 01/01/2011 to 28/07/2024, totaling 3332 observations. They were downloaded from the website www.investing.com.

The index prices were then converted into logarithmic returns $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1})$, where P_t denotes the index price and \ln corresponds to the natural logarithm.

3.2. Method

In this section, we will present the MF-DFA method according to Kantelhardt et al. [10]. MF-DFA consists of five distinct steps.

Consider a time series $X = (X(k))_{1 \leq k \leq N}$, representing a financial series, where N is the length of the series. It is assumed that this series has a compact support, meaning that $X(k) = 0$ for only a negligible fraction of the values.

Step 1: We determine the profile $Y = (Y(i))_{1 \leq i \leq N}$ of the series X defined by:

$$Y(i) = \sum_{k=1}^N (X(k) - \bar{X}) \quad (1)$$

where \bar{X} is the mean of the series X .

Step 2: For a given time-scale s , we divide the profile Y into $N_s = \text{Int}(N/s)$ non-overlapping segments of the same length s , where $\text{Int}(\cdot)$ represents the function that gives the integer part of a real number. Based on the recommendations of Peng et al. [11], $5 \leq s \leq N/4$ is traditionally selected. Since N is generally not a multiple of s , a short part at the end of the profile may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_s$ segments and there are two types of segmentation: for $1 \leq i \leq s$

- $Y((v-1)s + i)$ for $1 \leq v \leq N_s$.
- $Y((N-v-N_s)s + i)$ for $N_s + 1 \leq v \leq 2N_s$.

In each segment, we use the Ordinary Least Squares (OLS) method to properly fit the data with a local trend. In this study, we denote by $p_v^m(i)$ the fitting polynomial for the v -th segment: For $1 \leq v \leq 2N_S$

$$p_v^m(i) = \alpha_0^v + \alpha_1^v \cdot i + \dots + \alpha_m^v \cdot i^m \quad (2)$$

In the empirical study, the order m can be quadratic, cubic, or even of a higher order. Choosing an appropriate value of m can avoid overfitting the series.

After determining $p_v^m(i)$, we calculate the variances $F^2(v, s)$ for all time scales s and for $1 \leq v \leq 2N_S$. The variance $F^2(v, s)$ is defined by:

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^s [Y((v-1)s+i) - p_v^m(i)]^2 \quad (3)$$

for $1 \leq v \leq N_S$, and:

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^s [Y((N-v-Ns)s+i) - p_v^m(i)]^2 \quad (4)$$

for $N_S + 1 \leq v \leq 2N_S$.

Step 4: By averaging the variances over all segments, we obtain the fluctuation function $F_q(s)$ of order q defined by:

$$F_q(s) = \left[\frac{1}{2N_S} \sum_{i=1}^{2N_S} (F^2(v, s))^{\frac{q}{2}} \right]^{\frac{1}{q}} \quad (5)$$

for $q \neq 0$, and:

$$F_0(s) = \exp \left[\frac{1}{4N_S} \sum_{i=1}^{2N_S} \ln(F^2(v, s)) \right] \quad (6)$$

for $q = 0$.

The purpose of the MF-DFA procedure is primarily to determine the behavior of the fluctuation functions $F_q(s)$ as a function of the time-scale s for various values of q . To this end, steps 2 through 4 must be repeated for different time scales s .

Step 5: We analyze the multi-scale behavior of the fluctuation functions $F_q(s)$ by estimating the slope of the log-log plots of $F_q(s)$ versus s for different values of q . If the analyzed time series exhibits long-term correlation according to a power-law, such as fractal properties, the fluctuation function $F_q(s)$ will behave, for sufficiently large values of s , according to the following power-law scaling law:

$$F_q(s) \sim s^{H(q)} \quad (7)$$

In general, the exponent $H(q)$ can depend on q . To estimate the values of $H(q)$ for different values of q , we perform a semi-logarithmic regression of the time series $H(q)$ on the time series $F_q(s)$.

If stationary time series are dealt with, only the exponent $H(2)$ is obtained, which is identically equal to the standard Hurst exponent H . Therefore, the exponent $H(q)$ generalizes the Hurst exponent H and is commonly referred to as the generalized Hurst exponent. To distinguish between monofractal and multifractal time series, we can say that if $H(q) = H$ for all values of q , then the studied time series is monofractal; otherwise, $H(q)$ is a monotonically decreasing function of q , and the corresponding time series is multifractal.

It is well known that the generalized Hurst exponent $H(q)$ is directly related to the multifractal scaling exponent $\tau(q)$, commonly known as the Rényi exponent:

$$\tau(q) = q \cdot H(q) - 1 \quad (8)$$

It is clear that monofractal time series are characterized by a linear form for the Rényi exponent:

$$\tau(q) = q.H - 1 \quad (9)$$

where H is the Hurst exponent.

Another interesting way to characterize the multifractality of time series is to use the Hölder spectrum or the singularity spectrum $f(\alpha)$ of the Hölder exponent α . It is well known that the singularity spectrum $f(\alpha)$ is related to the Rényi exponent $\tau(q)$ through the Legendre transform:

$$\begin{cases} \alpha = \tau'(q) \\ f(\alpha) = q.\alpha - \tau(q) \end{cases} \quad (10)$$

where $\tau'(q)$ is the derivative of the function $\tau(q)$.

The richness of the multifractality can be determined by the width of the spectrum, $\Delta\alpha = \alpha_{max} - \alpha_{min}$. Thus, the wider the spectrum, the richer the multifractal behavior of the analyzed time series. We can easily deduce the relationship between the generalized Hurst exponent $H(q)$ and the singularity spectrum $f(\alpha)$:

$$\begin{cases} \alpha = h(q) + q.h'(q) \\ f(\alpha) = q.(\alpha - h(q)) + 1 \end{cases} \quad (11)$$

3.3. Sources of multifractality

Kantelhardt et al. [10] identified two primary sources of multifractality in a time series, long-term temporal correlations and heavy-tailed distributions. To determine how each source contributes to the overall multifractality, we can use two transformations on the original return series, namely the shuffling (random permutation) and the surrogation (phase randomization).

By shuffling the return series, the distribution of different moments is preserved, but long-term correlations are eliminated. After random permutation, the data have the same distribution but no temporal correlation or memory.

The surrogation helps isolate the contribution of long-term correlations to multifractality by randomly shifting the temporal phases of the data and disrupting these correlations while preserving their overall fluctuation behavior.

In the literature, there are various techniques for surrogation:

- Inverse Fast Fourier Transform (IFFT) (Proakis et al., [27]).
- Iterated Algorithm (iAAFT) (Schreiber and Schmitz, [28]).
- Statically Transformed Autoregressive Process (STAP) (Kugiumtzis, [29]).

In this study, we used two shuffling techniques, based on two functions “randperm” and “randi”. For the surrogation we applied the Inverse Fast Fourier Transform (IFFT) method.

4. Results and Discussion

In this section, we begin by applying the MF-DFA technique to examine the multifractal properties of the original logarithmic return series of the six Islamic indices, followed by an analysis to identify the sources of this multifractality.

4.1. Multi-scale behavior of the fluctuation functions

We analyzed the multi-scale behavior of the fluctuation functions $F_q(s)$ with respect to the time-scales s within the interval $S = [20:10:100,200:100:3000]$ for values of q in the interval $[-45:5:-5, -2.1:0.1:-0.1,0.1:0.1:2.1,5:5:45]$. By regressing $\text{Log}(F_q(s))$ on $\text{Log}(s)$, we obtain an estimation of $H(q)$:

$$\text{Log}(F_q(s)) \approx H(q).\text{Log}(s) \quad (12)$$

Figure 1 shows the logarithmic scale plots of $F_q(s)$ with respect to s for 9 values of q chosen from $\{-10, -5, -2, -1, 0, 1, 2, 5, 10\}$.

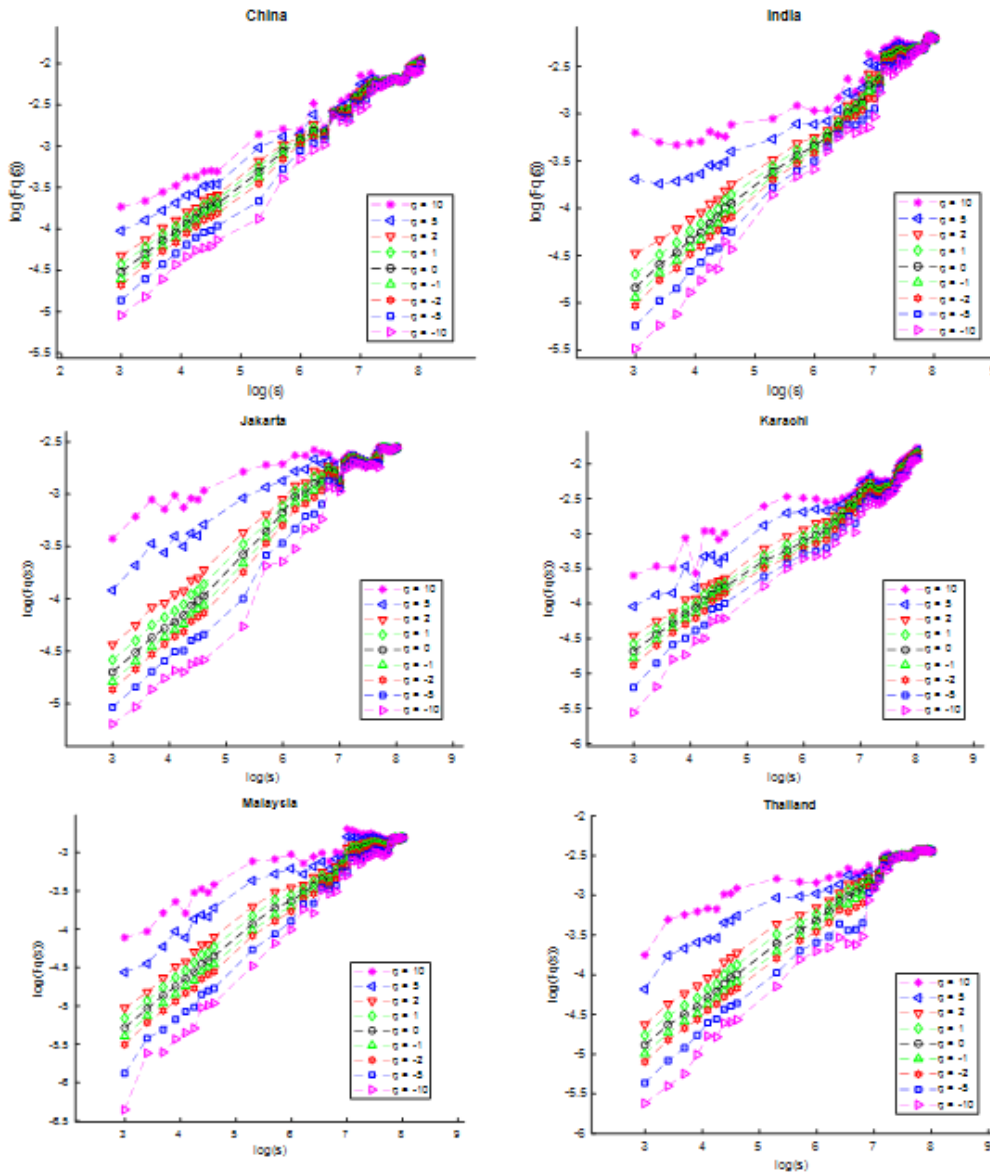


Figure 1.
 Fluctuation functions $\text{Log}\left(F_q^{XY}(s)\right)$ vs. $\text{Log}(s)$

4.2. Generalized Hurst Exponents $H(q)$

The generalized Hurst exponents $H(q)$ are given by the slopes of the lines obtained by least squares fitting $\text{Log}(s)$ on $\text{Log}\left(F_q(s)\right)$. Figure 2 shows the plots of the generalized Hurst exponents $H(q)$ as functions of $q \in [-45; 5; -5, -2.1; 0.1; -0.1, 0.1; 0.1; 2.1, 5; 5; 45]$.

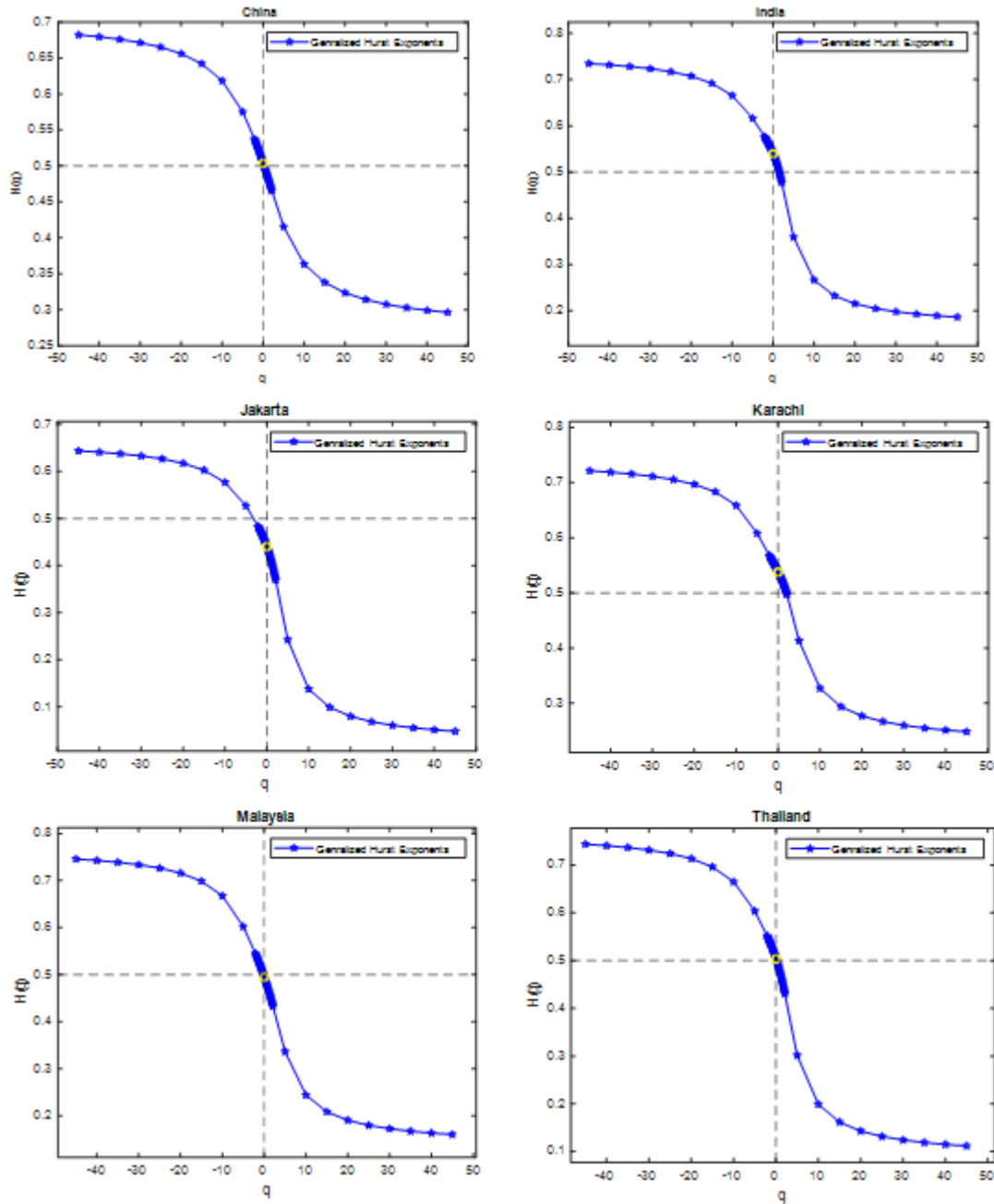


Figure 2. Generalized Hurst exponents $H(q)$ for q in the interval $[-45:5:-5, -2.1:0.1:-0.1,0.1:0.1:2.1,5:5:45]$.

As shown in the previous figure, as q increases from -45 to 45 , the generalized Hurst exponent $H(q)$ decreases non-linearly. This indicates that the six indices exhibit multifractal nature, suggesting a weak form inefficiency of the six Islamic stock markets.

The degree of multifractality in the returns series of the six indices could be measured by the difference between the smallest and largest values of $H(q)$:

$$\Delta H = H_{max} - H_{min} = H(q_{min}) - H(q_{max}) \tag{13}$$

For a monofractal series, $\Delta H = 0$ is obtained. The larger ΔH is, the higher the degree of multifractality.

Table 1 presents the degree of multifractality for the six indices, ordered in decreasing order based on the generalized Hurst exponent.

Table 1.
Degrees of multifractality in decreasing order based on the generalized Hurst exponent.

Rank	Index	ΔH
1	Thailand	0.632
2	Jakarta	0.596
3	Malaysia	0.585
4	India	0.549
5	Karachi	0.473
6	China	0.386

As we can see, Thailand presents the highest degree of multifractality, followed by Jakarta, Malaysia, India, Karachi and China. This suggests that the economic systems of these countries exhibit varying levels of complexity and instability. Thailand's high degree of multifractality indicates that its economic system is maybe highly complex, with fluctuations across multiple scales or factors. This could suggest that Thailand's economy experiences greater unpredictability or volatility, making it more sensitive to changes in various economic drivers such as market conditions, government policies, or external shocks. Jakarta, being second in multifractality, likely exhibits a slightly lower level of complexity but still shows substantial variability in its economic patterns, which could reflect dynamic growth or challenges in managing economic stability. Malaysia, India, Karachi, and China follow in decreasing order of multifractality, indicating that their economies are more stable or less volatile compared to Thailand and Jakarta. However, they may still experience some degree of complexity or variability, possibly due to differing market structures, policy environments, or external economic factors.

4.3. Rényi Exponent $\tau(q)$

Figure 3 shows the plots of the Rényi exponents $\tau(q)$ as functions of the variable $q \in [-45:5: -5, -2.1: 0.1: -0.1, 0.1: 0.1: 2.1, 5: 5: 45]$.

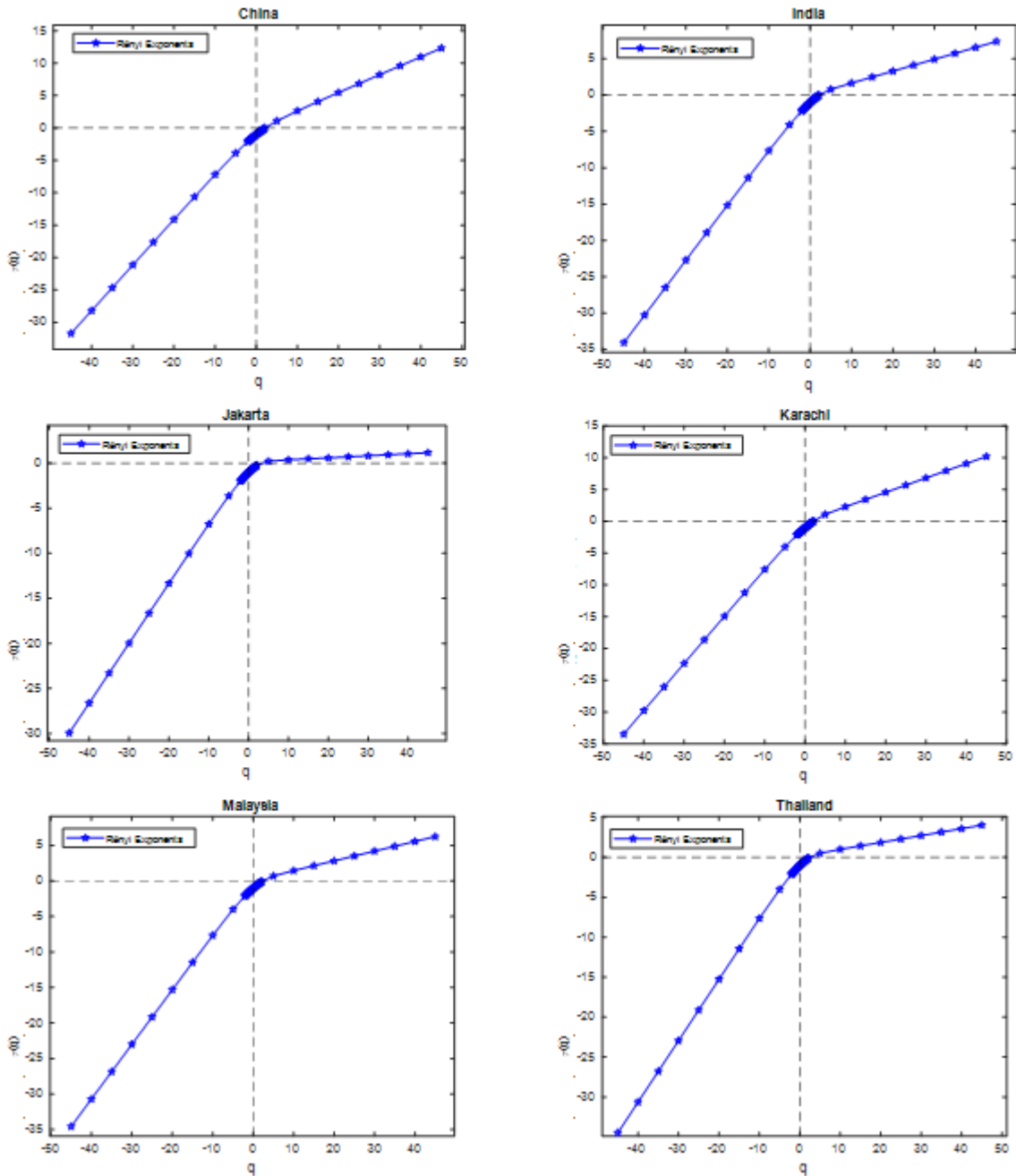


Figure 3. Rényi exponents $\tau(q)$ for $q \in [-45:5; -5, -2.1:0.1; -0.1,0.1:0.1; 2.1,5:5; 45]$.

As shown in the previous figure, it can be observed that as q increases from -45 to 45 , the Rényi exponent $\tau(q)$ increases non-linearly. This indicates that the six indices exhibit multifractal nature, suggesting a weak form inefficiency in the six Islamic stock markets.

4.4. Hölder Singularity Spectrum $f(\alpha)$

Another interesting way to characterize the multifractality of time series is to use the singularity spectrum $f(\alpha)$. Figure 4 shows the plots of the singularity spectrum functions $f(\alpha)$ for the six indices.

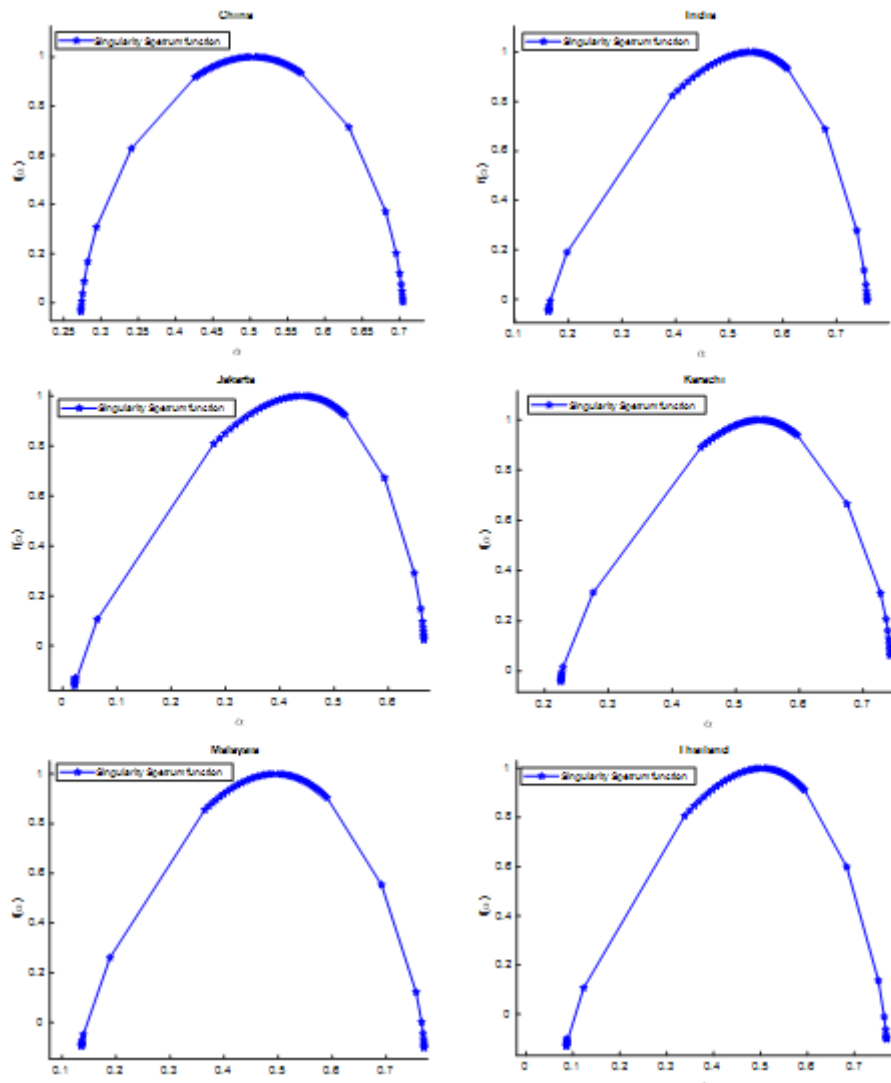


Figure 4. Singularity spectrum functions $f(\alpha)$ for the six indices.

We can observe in this figure that the curves of the singularity spectrum functions $f(\alpha)$ have an inverted parabolic shape. This indicates that the six indices exhibit multifractal nature, suggesting a weak form inefficiency in the six Islamic stock markets. Recall that for monofractal time series, the curve of the singularity spectrum theoretically reduces to a single point $\alpha = H$ with $f(\alpha) = 1$.

We can measure the degree of multifractality by calculating the width of the spectrum, given by:

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{14}$$

Table 2 presents the degree of multifractality for the six indices in decreasing order.

Table 2.
Degrees of multifractality in decreasing order based on the singularity spectrum.

Rank	Index	$\Delta\alpha$
1	Thailand	0.682
2	Jakarta	0.644
3	Malaysia	0.634
4	India	0.594
5	Karachi	0.517
6	China	0.431

We observe the same ranking of the degrees of multifractality as those derived from the generalized Hurst exponent.

4.5. Source of Multifractality for the Six Islamic Indices

As previously noted, there are two different sources of multifractality in a time series, long-term temporal correlations and the heavy tails distributions. To determine how each source contributes to the overall multifractality, we will use the shuffling and the surrogation transformations on the original geometric return series.

In this study, two shuffling techniques are applied, namely “randperm” and “randi”. For the surrogation, the Inverse Fast Fourier Transform (IFFT) method is applied.

Figures 5 and 6 compare the curves of the generalized Hurst exponent $H(q)$ and the curves of the singularity spectrum $f(\alpha)$ for the six original series with those of the surrogate and the shuffled series.

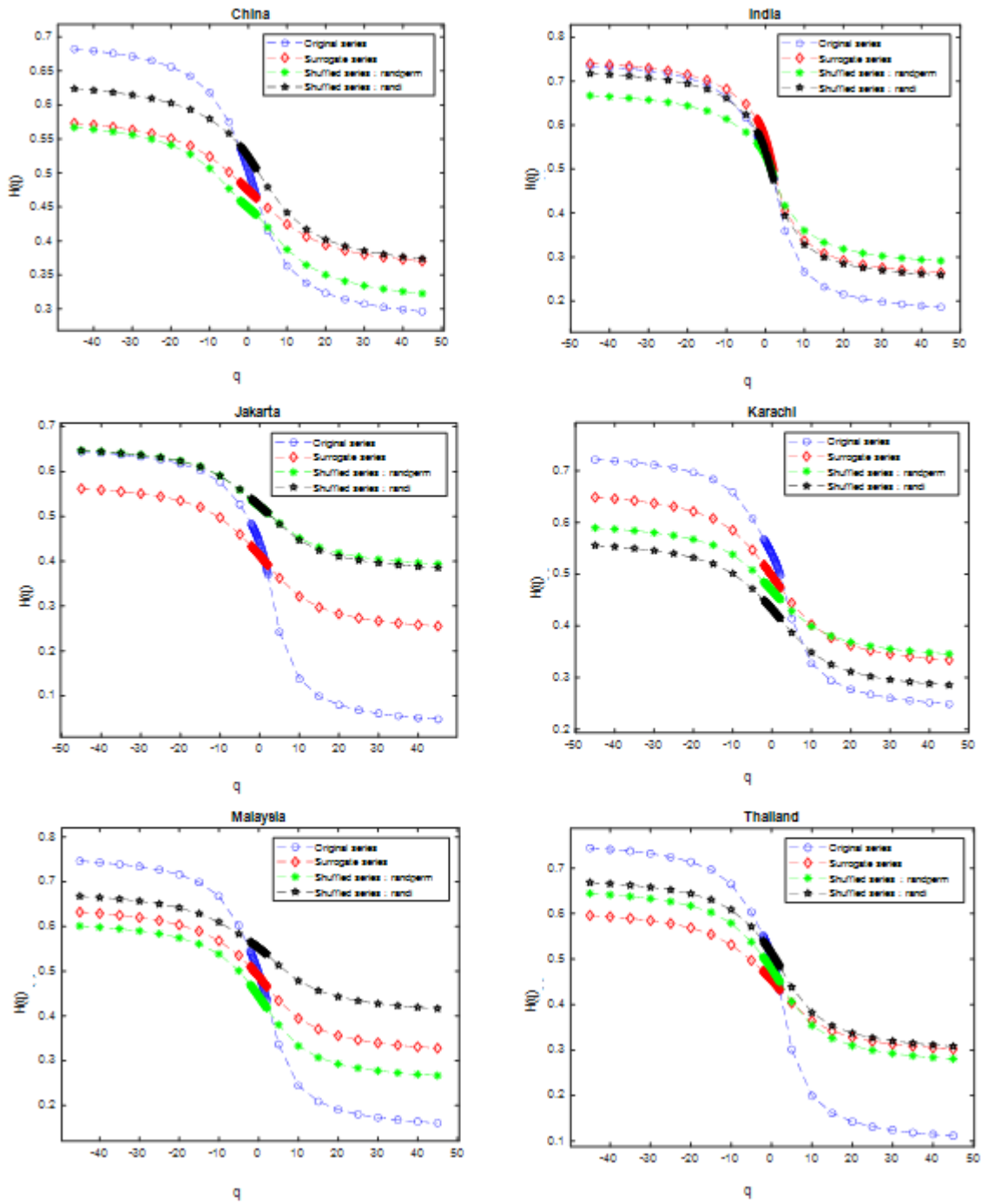


Figure 5. Generalized Hurst exponent $H(q)$ vs. q for original, surrogate and shuffled.

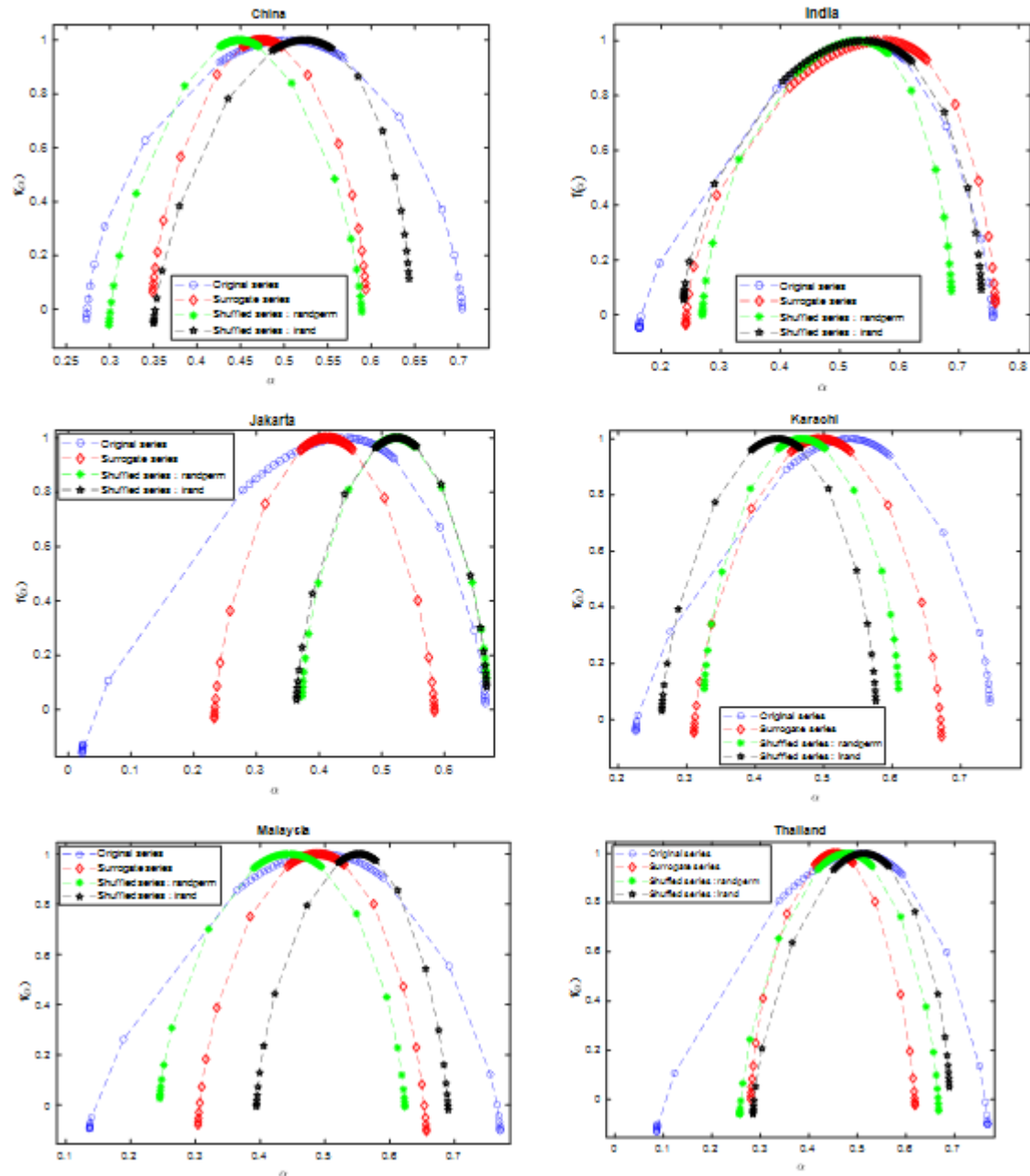


Figure 6.
Singularity spectra $f(\alpha)$ vs. α for original, surrogate and shuffled.

To precisely measure how much multifractality has been reduced, we calculate the values of $\Delta H = H_{max} - H_{min}$ and $\Delta\alpha = \alpha_{max} - \alpha_{min}$ for the six indices.

The MF-DCCA program has been run 100 times for each index, and each time different results for the original series and the two shuffled series were obtained, the results for the surrogate series remained consistent. This variability is due to the algorithms generating the surrogate and shuffled series using random permutations. However, in all 100 simulations, the ΔH and $\Delta\alpha$ of the original

series are greater than the ΔH and $\Delta\alpha$ of the surrogate and two shuffled series. The results are presented in Table 3.

Table 3.

Degrees of multifractality of original, surrogate and shuffled series based ΔH and $\Delta\alpha$.

Index	Original		Surrogate		Shuffled-randperm		Shuffled-randi	
	ΔH	$\Delta\alpha$	ΔH	$\Delta\alpha$	ΔH	$\Delta\alpha$	ΔH	$\Delta\alpha$
China	0.386	0.431	0.203	0.244	0.244	0.290	0.250	0.293
India	0.549	0.594	0.476	0.520	0.376	0.418	0.459	0.500
Jakarta	0.596	0.644	0.306	0.351	0.253	0.294	0.261	0.303
Karachi	0.473	0.517	0.315	0.362	0.244	0.283	0.270	0.312
Malaysia	0.585	0.634	0.304	0.353	0.334	0.378	0.251	0.296
Thailand	0.632	0.682	0.294	0.339	0.364	0.410	0.360	0.405

The results indicate that $\Delta H_{Original} > \Delta H_{Surrogate}$, $\Delta\alpha_{original} > \Delta\alpha_{Surrogate}$, $\Delta H_{Original} > \Delta H_{Shuffled}$ and $\Delta\alpha_{original} > \Delta\alpha_{Shuffled}$ for the six indices, as confirmed in the previous table. This indicates that the multifractality of the six indices has been reduced by both the surrogation and the shuffling transformations. We conclude that both long-term correlation and heavy-tailed distributions contribute to the multifractal behavior of the six indices.

In summary, based on generalized Hurst exponents and singularity spectrum, we found that both long-term correlations and heavy-tailed distributions contribute to the multifractal behavior of the six Islamic stock markets in the Asia-Pacific region.

This finding has significant implications in the six Islamic stock markets. The reduction of multifractality through surrogation and shuffling transformations implies that long-term correlations and heavy-tailed distributions are crucial for understanding the risk profiles of the indices. For risk managers, this highlights the need to consider both long-term dependencies and extreme events when assessing market risks. A model that incorporates these factors could better account for tail risks and extreme fluctuations, leading to more robust risk management strategies. For investors, the reduction in multifractality could suggest that certain strategies relying on more predictable correlations may be more effective in times of reduced market complexity. This could be valuable in constructing portfolios and devising investment strategies that aim to mitigate risk during periods of lower market complexity, where the long-term dependencies and extreme events may be less impactful. For regulators, these findings emphasize the importance of considering both long-term correlations and the impact of extreme market events. By identifying the sources of multifractality, policymakers can better understand the systemic risks present in the market, particularly in terms of how shocks in one market may propagate across others. This insight could guide the development of regulations to mitigate systemic risk and improve market stability.

4.6. Discussion

In comparing the results of the current study with previous literature, several key points emerge. The current research on Islamic stock markets in the Asia-Pacific region, using the MF-DFA method, mirrors many findings from global studies in demonstrating that multifractality is a widespread characteristic in financial markets, highlighting inefficiencies that traditional models may fail to capture.

Previous studies, such as those by Norouzzadeh and Rahmani [12] and Xinsheng et al. [13], also revealed that multifractality stems from both long-term correlations and fat-tailed distributions. These studies, similar to the present work, found that multifractality challenges the assumption of market efficiency. The findings in this paper, where long-term correlations and heavy-tailed distributions are identified as key contributors to market inefficiencies, echo the results of Wang et al. [16] and El Alaoui and Benbachir [15], who also identified these two elements as significant drivers of multifractal behavior in stock markets.

The study also aligns with Rui et al. [17] and Hasan and Mohammad [18], where multifractality was observed across various markets and during crises, underscoring that market inefficiencies can persist even during periods of market stress. The presence of multifractality in the Asia-Pacific Islamic stock markets reinforces the idea that such inefficiencies are not confined to specific regions or asset classes but are a global phenomenon. This is consistent with findings by Faheem et al. [24] and Raza et al. [26], who observed varying degrees of multifractality across different markets and sectors.

Importantly, the current study contributes to the literature by focusing specifically on Islamic stock markets in the Asia-Pacific region, providing a regional perspective on how multifractality manifests in these markets. The results support the findings from Tiwari et al. [19] and Hong-Yong and Tong-Tong [20], which highlight the presence of multifractality in sectoral indices and the variability of fractal characteristics across time scales.

In summary, the findings from the current study, which confirm the multifractal nature of Islamic stock markets in the Asia-Pacific region, align closely with previous research across various markets and time periods. The results provide further evidence of the applicability of MF-DFA in detecting market inefficiencies and understanding the complexities that traditional models may overlook. This comparison with the existing literature highlights the value of multifractal analysis as a tool for assessing market behavior, especially in identifying sources of inefficiency and forecasting future market movements.

5. Conclusion

In conclusion, this paper provides a thorough assessment of the informational efficiency of seven Islamic stock markets in the Asia-Pacific region by focusing on their multifractal characteristics. The study has successfully met its two primary objectives: capturing and quantifying the degree of multifractality and identifying its sources within these markets.

Through the application of the MF-DFA method, based on generalized Hurst exponents, Rényi exponents, and the singularity spectrum, the research reveals that all six Islamic markets exhibit multifractal properties. This finding is significant as it suggests a departure from the Efficient Market Hypothesis, which posits that markets are fully efficient and that asset prices reflect all available information. The observed multifractality implies that these markets do not adhere to this hypothesis, highlighting inefficiencies where historical price patterns and long-term trends can potentially be leveraged to gain insights into future market movements.

Moreover, we employ surrogation and shuffling techniques to further investigate the sources of multifractality. The results indicate that both long-term correlations and heavy-tailed distributions are key factors contributing to the observed multifractal behavior. Together, these elements challenge the notion of market randomness and efficiency, indicating that the markets under study exhibit complex dynamics that cannot be fully explained by traditional models of market efficiency.

The results of this study lead to several practical implications for investors and policymakers. Investors should consider incorporating multifractal analysis into their investment strategies. Traditional models may not fully capture the complexities observed in these markets. Employing methods that account for long-term correlations and extreme events could enhance predictive accuracy and risk management. Given the presence of multifractality and market inefficiencies, diversification can help mitigate risk. Investors should consider spreading their investments across different assets and markets to reduce exposure to any single market's idiosyncrasies. Investors should remain vigilant about long-term trends and market patterns. Tools and strategies that focus on capturing long-term correlations and understanding heavy-tailed distributions might provide valuable insights into potential market movements. Policymakers should focus on improving market transparency to address inefficiencies. Ensuring that all market participants have access to relevant information can help reduce the effects of multifractality and improve overall market efficiency. Given the multifractal nature of these markets, targeted regulatory measures may be necessary to address specific market inefficiencies. This could include regulations that address extreme events or improve the handling of long-term market dependencies. Policymakers should consider measures to enhance market stability, such as

stress-testing financial systems for extreme events and long-term trends. This can help in mitigating the impact of market inefficiencies and ensuring a more resilient financial environment.

Future research should delve deeper into the specific factors driving multifractality in these markets. Understanding the underlying mechanisms can provide more targeted insights and improve the accuracy of financial models.

Overall, this study advances our understanding of the informational efficiency of Islamic stock markets in the Asia-Pacific region, offering a comprehensive analysis of their multifractal nature and highlighting areas for future investigation and improvement.

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