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The Helmholtzian Operator and Maxwell-Cassano Equations of an Electromagnetic-nuclear Field

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Abstract: This article demonstrates that when acting on a four-vector doublet, the Helmholtzian may be factored into two 4×4/8×8 differential matrices, resulting in a four-vector doublet Klein-Gordon equation with source. This factorization enables yielding of a mass-generalized set of Maxwell's equations.

Keywords: *Helmholtzian operator, Maxwell-Cassano equations, Electromagnetic-nuclear Field, Maxwell's equations, Klein-Gordon equation, Dirac equation, d'Alembertian, Helmholtz.*

1. Introduction

The He1mholtz differential equation [1-3] is a linear second order differential equation, generalization of the wave equation. A Helmholtzian operator is a linear second order differential operator, typically in four independant doublet-variables; a generalization of the d'Alembertian operator [4] where it's additional constant vanishes. When the time-independant version (in three independent (space) variables acting on a function or vector vanishes, the resulting equation is called the Helmholtz's equation. In four dimensions this equation is referred to as the Klein-Gordon equation [5-9] (with imaginary constant).

(Because the Klein-Gordon equation is the four-dimensional generalization of the three-dimensional Helmholtz equation; and each, of course may be generalized to higher dimensions, I have chosen to denote the operator with the Helmholtz designation).

This article demonstrates that when acting on a four-vector doublet, the Helmholtzian may be factored into two 4×4/8 × 8 differentials matrices in two distinct ways, as follows:

$$\mathbf{J} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\square - |\mathrm{m}|^2) f^1 \\ (\square - |\mathrm{m}|^2) f^2 \\ (\square - |\mathrm{m}|^2) f^3 \\ (\square - |\mathrm{m}|^2) f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \qquad (1)$$

Where:

$$D_i^+ \equiv (\partial_i + m_i),\ D_i^- \equiv (\partial_i - m_i),\ \partial_i \equiv \frac{\partial}{\partial x^i},\ m_i \quad \text{Constants} \qquad (2)$$

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$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix},\ D_i^{\Updownarrow} \equiv \begin{pmatrix} D_i^- & 0 \\ 0 & D_i^+ \end{pmatrix},\ D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix},\ D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_i^- & 0 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \partial_0 & \partial_3 & -\partial_2 & \partial_1 \\ -\partial_3 & \partial_0 & \partial_1 & \partial_2 \\ \partial_2 & -\partial_1 & \partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} \partial_0 & -\partial_3 & \partial_2 & \partial_1 \\ \partial_3 & \partial_0 & -\partial_1 & \partial_2 \\ -\partial_2 & \partial_1 & \partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} \Box f^1 \\ \Box f^2 \\ \Box f^3 \\ \Box f^0 \end{pmatrix} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \equiv \mathbf{J} \tag{4}$$

$$\begin{pmatrix} -\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} -\partial_0 & -\partial_3 & \partial_2 & -\partial_1 \\ \partial_3 & -\partial_0 & -\partial_1 & -\partial_2 \\ -\partial_2 & \partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} \Box f^1 \\ \Box f^2 \\ \Box f^3 \\ \Box f^0 \end{pmatrix} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \equiv \mathbf{J} \tag{5}$$

2. Discussion

It doesn't take much more than a cursory look to see that this Helmholtzian operator and factorization is a generalization of the d'Alembertian operator and its factorization [4].

The four-vector-doublet Klein-Gordon equation may be written as a matrix product. This, thus, when operated on a four-vector-doublet, gives a matrix product definition for the Helmholtzian operator. (The scalar Klein-Gordon equation is a special case of this four-vector-doublet version, where there are three restrictions on the four component-doublets of the potential A , leaving the single independent function-pair.)(Note that the Dirac equation [5-9] is a set of equations on a scalar doublet; so there is, in this way, Helmholtzian factorization consistency.)(And, there is a deeply intimate relationship with the Dirac equation).

Another great thing about using this description of the Helmholtzian is that if the matrix is applied to the column vector, the result may be expressed in terms of generalized E and B vector components associated with generalized electromagnetic field (with the appropriate definitions of A0 and x0)(just as with the d'Alembertian operator). And, when the final matrix is applied, the result is mass-generalized.

Maxwell's inhomogeneous field equations [10-15] (with the gauge fixing term)(because the homogeneous field equations are identities, which actually appear as such in the final computation by all those terms canceling each other out). So, this is an incredibly compact way of writing both the Klein-Gordon equation and mass-generalized Maxwell's equations of an electromagnetic-nuclear field [10-15].

As shown for the d'Alembertian operator in (4):

$$\mathbf{E} = \mathbf{u}_\mu(-\partial_0 f^\mu - \partial_\mu f^0),\quad (\mu \in N, 1 \le \mu \le 3) \tag{6}$$

$$\mathbf{B} = \mathbf{u}_\mu (-1)^{\mu+1}[\partial_{2-T_0(\mu)} f^{3-T_0(\mu-1)} - \partial_{3-T_0(\mu-1)} f^{2-T_0(\mu)}] \quad , \quad (\mu \in N, 1 \le \mu \le 3)$$

$$T_0(j) \equiv \frac{1}{2}\left(j - 1 + \delta^1_{(-1)^j}\right), (j \in N) \tag{7}$$

$$\begin{pmatrix} -\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} -\partial_0 & -\partial_3 & \partial_2 & -\partial_1 \\ \partial_3 & -\partial_0 & -\partial_1 & -\partial_2 \\ -\partial_2 & \partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} \Box f^1 \\ \Box f^2 \\ \Box f^3 \\ \Box f^0 \end{pmatrix} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \equiv \mathbf{J} \tag{8}$$

$$= \begin{pmatrix} -\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} B^1 + E^1 \\ B^2 + E^2 \\ B^3 + E^3 \\ -\nabla^* \Box \mathbf{f} \end{pmatrix} = \begin{pmatrix} -\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \left[(\mathbf{B} + \mathbf{E}) - (\nabla^* \Box\ \mathbf{f})\right] \tag{9}$$

Where:

$\mathbf{f} \equiv \mathbf{u}_\mu f^\mu$	$\mathbf{f}^* \equiv (-1)^{\delta_0^\mu}\, \mathbf{u}_\mu f^\mu$
$\nabla \equiv \mathbf{u}_\mu \partial_\mu$	$\nabla^* \equiv (-1)^{\delta_0^\mu}\, \mathbf{u}_\mu \partial_\mu$
$\nabla.\mathbf{f} = \partial_\mu f^\mu$	$\nabla.\mathbf{f}^* = (-1)^{\delta_0^\mu}\, \partial_\mu f^\mu = \nabla^*.\mathbf{f}$
$\nabla \Box\ \mathrm{f} = \mathbf{u}_0(\nabla.\mathbf{f})$	$\nabla^* \Box\ \mathrm{f} = \mathbf{u}_0(\nabla^*.\,\mathbf{f})$

$$
= \begin{pmatrix}
-\partial_0(B^1+E^1) + \partial_3(B^2+E^2) - \partial_2(B^3+E^3) - \partial_1(-\nabla^*\Box \mathrm{f}) \\
-\partial_3(B^1+E^1) - \partial_0(B^2+E^2) + \partial_1(B^3+E^3) - \partial_2(-\nabla^*\Box \mathrm{f}) \\
\partial_2(B^1+E^1) - \partial_1(B^2+E^2) - \partial_0(B^3+E^3) - \partial_3(-\nabla^*\Box \mathrm{f}) \\
-\partial_1(B^1+E^1) - \partial_2(B^2+E^2) - \partial_3(B^3+E^3) + \partial_0(-\nabla^*\Box \mathrm{f})
\end{pmatrix} \tag{10}
$$

$$
= -\begin{pmatrix}
(\partial_0 B^1 + \partial_2 E^3 - \partial_3 E^2) + (\partial_0 E^1 + \partial_2 B^3 - \partial_3 B^2) - \partial_1(\nabla^*\Box \mathrm{f}) \\
(\partial_0 B^2 - \partial_1 E^3 + \partial_3 E^1) + (\partial_0 E^2 - \partial_1 B^3 + \partial_3 B^1) - \partial_2(\nabla^*\Box \mathrm{f}) \\
(\partial_0 B^3 + \partial_1 E^2 - \partial_2 E^1) + (\partial_0 E^3 + \partial_1 B^2 - \partial_2 B^1) - \partial_3(\nabla^*\Box \mathrm{f}) \\
(\partial_1 B^1 + \partial_2 B^2 + \partial_3 B^3) + (\partial_1 E^1 + \partial_2 E^2 - \partial_3 E^3) + \partial_0(\nabla^*\Box \mathrm{f})
\end{pmatrix} \tag{11}
$$

$$
= -\begin{pmatrix}
(\partial_0 E^1 + \partial_2 B^3 - \partial_3 B^2) - \partial_1(\nabla^*\Box \mathrm{f}) \\
(\partial_0 E^2 - \partial_1 B^3 + \partial_3 B^1) - \partial_2(\nabla^*\Box \mathrm{f}) \\
(\partial_0 E^3 + \partial_1 B^2 - \partial_2 B^1) - \partial_3(\nabla^*\Box \mathrm{f}) \\
(\partial_1 E^1 + \partial_2 E^2 + \partial_3 E^3) + \partial_0(\nabla^*\Box \mathrm{f})
\end{pmatrix} \tag{12}
$$

and similarly for the other factorization.
Similarly, for the Helmholtzian operator factorization:

$$
\begin{aligned}
\mathbf{E} &= \mathbf{w}^{4;1}\left(-D_0^{\Updownarrow} f^1 - D_1 f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Updownarrow} f^2 - D_2 f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Updownarrow} f^3 - D_3 f^0\right) \\
\mathbf{B} &= \mathbf{w}^{4;1}\left(D_2 f^3 - D_3 f^2\right) + \mathbf{w}^{4;2}\left(-D_1 f^3 + D_3 f^1\right) + \mathbf{w}^{4;3}\left(D_1 f^2 - D_2 f^1\right) \\
\mathbf{E}_{\Updownarrow} &= \mathbf{w}^{4;1}\left(-D_0^{\Leftrightarrow\Updownarrow} f^1 - D_1^{\Leftrightarrow} f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Leftrightarrow\Updownarrow} f^2 - D_2^{\Leftrightarrow} f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Leftrightarrow\Updownarrow} f^3 - D_3^{\Leftrightarrow} f^0\right) \\
\mathbf{B}_{\Updownarrow} &= \mathbf{w}^{4;1}\left(D_2^{\Leftrightarrow} f^3 - D_3^{\Leftrightarrow} f^2\right) + \mathbf{w}^{4;2}\left(-D_1^{\Leftrightarrow} f^3 + D_3^{\Leftrightarrow} f^1\right) + \mathbf{w}^{4;3}\left(D_1^{\Leftrightarrow} f^2 - D_2^{\Leftrightarrow} f^1\right);\ (13)
\end{aligned} \tag{13}
$$

Where:

$\mathbf{f} \equiv \mathbf{w}^{4;1} f^\mu$	$f^\mu \equiv \begin{pmatrix} f_+^\mu \\ f_-^\mu \end{pmatrix}$

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$$\mathbf{J}=\begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix}=\begin{pmatrix} (\Box\text{-}|\mathrm{m}|^2)\, f^1 \\ (\Box\text{-}|\mathrm{m}|^2)\, f^2 \\ (\Box\text{-}|\mathrm{m}|^2)\, f^3 \\ (\Box\text{-}|\mathrm{m}|^2)\, f^0 \end{pmatrix}=$$

$$=\begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix}\begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix}\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}= \quad (14)$$

$$=\begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix}\begin{pmatrix} B_{\Updownarrow}^1+E^1 \\ B_{\Updownarrow}^2+E^2 \\ B_{\Updownarrow}^3+E^3 \\ -\overset{m*}{\nabla}_{\Updownarrow} \Box \mathrm{f} \end{pmatrix}= \quad (15a)$$

$$=\begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix}\begin{pmatrix} B_{\Updownarrow}^1-E^1 \\ B_{\Updownarrow}^2-E^2 \\ B_{\Updownarrow}^3-E^3 \\ \overset{m}{\nabla}_{\Updownarrow} \Box \mathrm{f}^* \end{pmatrix}= \quad (15b)$$

$$=\begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix}\left[(\mathbf{B}_{\Updownarrow}-\mathbf{E})+(\overset{m}{\nabla}_{\Updownarrow} \Box\ \mathrm{f}^*)\right] \quad (16)$$

Where:

$\mathbf{f} \equiv \mathbf{w}^{4;1} f^{\mu}$	$\mathbf{f}^* \equiv (-1)^{\delta_0^{\mu}}\ \mathbf{w}^{4;1} f^{\mu}$
$f^{\mu} \equiv \begin{pmatrix} f_+^{\mu} \\ f_-^{\mu} \end{pmatrix}$	$f_{\Updownarrow}^{\mu} \equiv \begin{pmatrix} f_+^{\mu} \\ f_-^{\mu} \end{pmatrix}$
$\overset{m}{\nabla} \equiv \mathbf{w}^{4;1} D_1 + \mathbf{w}^{4;2} D_2 + \mathbf{w}^{4;3} D_3 + \mathbf{w}^{4;0} D_0^{\Updownarrow}$	$\overset{m}{\nabla}_{\Updownarrow} \equiv \mathbf{w}^{4;1} D_1^{\Updownarrow} + \mathbf{w}^{4;2} D_2^{\Updownarrow} + \mathbf{w}^{4;3} D_3^{\Updownarrow} + \mathbf{w}^{4;0} D_0$
$\overset{m}{\nabla}_{\Updownarrow} \,.\, \mathbf{f}^* = D_1^{\Updownarrow} f^1 + D_2^{\Updownarrow} f^2 + D_3^{\Updownarrow} f^3 - D_0 f^0$	$\overset{m}{\nabla}_{\Updownarrow} \Box\ \mathbf{f}^* = \mathbf{w}^{4;0}\left(D_0 f^0 - D_1^{\Updownarrow} f^1 - D_2^{\Updownarrow} f^2 - D_3^{\Updownarrow} f^3\right)$
$\overset{m}{\nabla} \times \mathbf{f} = \mathbf{w}^{4;1}(D_0^{\Updownarrow} f^1 - D_1 f^0 + D_2 f^3 - D_3 f^2) +$ $+ \mathbf{w}^{4;2}(D_0^{\Updownarrow} f^2 - D_1 f^3 - D_2 f^0 + D_3 f^1) +$ $+ \mathbf{w}^{4;3}(D_0^{\Updownarrow} f^3 - D_1 f^2 - D_2 f^1 - D_3 f^0).$	

$$
=\begin{pmatrix}
D_0(B^1_{\Updownarrow}-E^1)+D_3^{\Leftrightarrow}(B^2_{\Updownarrow}-E^2)-D_2^{\Leftrightarrow}(B^3_{\Updownarrow}-E^3)+D_1\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\
-D_3^{\Leftrightarrow}(B^1_{\Updownarrow}-E^1)+D_0(B^2_{\Updownarrow}-E^2)+D_1^{\Leftrightarrow}(B^3_{\Updownarrow}-E^3)+D_2\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\
D_2^{\Leftrightarrow}(B^1_{\Updownarrow}-E^1)-D_1^{\Leftrightarrow}(B^2_{\Updownarrow}-E^2)+D_0(B^3_{\Updownarrow}-E^3)+D_3\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\
D_1^{\Updownarrow}(B^1_{\Updownarrow}-E^1)+D_2^{\Updownarrow}(B^2_{\Updownarrow}-E^2)+D_3^{\Updownarrow}(B^3_{\Updownarrow}-E^3)-D_0^{\Updownarrow}\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)
\end{pmatrix}
\tag{17}
$$

$$
=\begin{pmatrix}
(D_0B^1_{\Updownarrow}+D_2^{\Leftrightarrow}E^3-D_3^{\Leftrightarrow}E^2)-(D_0E^1+D_2^{\Leftrightarrow}B^3_{\Updownarrow}-D_3^{\Leftrightarrow}B^2_{\Updownarrow})+D_1\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\
(D_0B^2_{\Updownarrow}-D_1^{\Leftrightarrow}E^3+D_3^{\Leftrightarrow}E^1)-(D_0E^2-D_1^{\Leftrightarrow}B^3_{\Updownarrow}+D_3^{\Leftrightarrow}B^1_{\Updownarrow})+D_2(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*)\\
(D_0B^3_{\Updownarrow}+D_1^{\Leftrightarrow}E^2-D_2^{\Leftrightarrow}E^1)-(D_0E^3+D_1^{\Leftrightarrow}B^2_{\Updownarrow}-D_2^{\Leftrightarrow}B^1_{\Updownarrow})+D_3(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*)\\
(D_1^{\Updownarrow}B^1_{\Updownarrow}+D_2^{\Updownarrow}B^2_{\Updownarrow}+D_3^{\Updownarrow}B^3_{\Updownarrow})-(D_1^{\Updownarrow}E^1+D_2^{\Updownarrow}E^2+D_3^{\Updownarrow}E^3)-D_0^{\Updownarrow}(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*)
\end{pmatrix}
\tag{18}
$$

$$
=-\begin{pmatrix}
(D_0E^1+D_2^{\Leftrightarrow}B^3_{\Updownarrow}-D_3^{\Leftrightarrow}B^2_{\Updownarrow})-D_1(\overset{m}{\nabla}_1\Box\mathbf{f}^*)\\
(D_0E^2-D_1^{\Leftrightarrow}B^3_{\Updownarrow}+D_3^{\Leftrightarrow}B^1_{\Updownarrow})-D_2(\overset{m}{\nabla}_1\Box\mathbf{f}^*)\\
(D_0E^3+D_1^{\Leftrightarrow}B^2_{\Updownarrow}-D_2^{\Leftrightarrow}B^1_{\Updownarrow})-D_3(\overset{m}{\nabla}_1\Box\mathbf{f}^*)\\
(D_1^{\Updownarrow}E^1+D_2^{\Updownarrow}E^2+D_3^{\Updownarrow}E^3)+D_0^{\Updownarrow}(\overset{m}{\nabla}_1\Box\mathbf{f}^*)
\end{pmatrix}
\tag{19}
$$

and similar for the other factorization.

Additionally, the above process may be generalized for any dimension power of two, thus generalizing the Helmholtzian (and, thus the d'Alembertian) as well as the mass-generalized Maxwell's equations to any dimension power of two (using the weighted matrix product to construct suitable algebras and doing the same analysis on it).

Comparing to the above [12] d'Alembertian factorization, a mass-generalization of Maxwell's equations is a clear result.

However, they are in a form beyond standard notation; so expanding them is in order.

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$$
\left(\Box-|m|^2\right)\begin{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1\end{pmatrix}\\ \begin{pmatrix} f_+^2 \\ f_-^2\end{pmatrix}\\ \begin{pmatrix} f_+^3 \\ f_-^3\end{pmatrix}\\ \begin{pmatrix} f_+^0 \\ f_-^0\end{pmatrix}\end{pmatrix}=\begin{pmatrix}\begin{pmatrix} J_+^1 \\ J_-^1\end{pmatrix}\\ \begin{pmatrix} J_+^2 \\ J_-^2\end{pmatrix}\\ \begin{pmatrix} J_+^3 \\ J_-^3\end{pmatrix}\\ \begin{pmatrix} J_+^0 \\ J_-^0\end{pmatrix}\end{pmatrix}=\mathbf{J}=
$$

$$
\begin{pmatrix}
\begin{pmatrix}
(\partial_0+m_0)E_+^1+(\partial_2-m_2)B_+^3-(\partial_3-m_3)B_+^2+ \\
\quad -(\partial_1+m_1)[(\partial_1-m_1)f_+^1+(\partial_2-m_2)f_+^2+(\partial_3-m_3)f_+^3-(\partial_0+m_0)f_+^0] \\
(\partial_0-m_0)E_-^1+(\partial_2+m_2)B_-^3-(\partial_3+m_3)B_-^2+ \\
\quad -(\partial_1-m_1)[(\partial_1+m_1)f_-^1+(\partial_2+m_2)f_-^2+(\partial_3+m_3)f_-^3-(\partial_0-m_0)f_-^0]
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0+m_0)E_+^2+(\partial_1-m_1)B_+^3-(\partial_3-m_3)B_+^1+ \\
\quad -(\partial_2+m_2)[(\partial_1-m_1)f_+^1+(\partial_2-m_2)f_+^2+(\partial_3-m_3)f_+^3-(\partial_0+m_0)f_+^0] \\
(\partial_0-m_0)E_-^2+(\partial_1+m_1)B_-^3-(\partial_3+m_3)B_-^1+ \\
\quad -(\partial_2-m_2)[(\partial_1+m_1)f_-^1+(\partial_2+m_2)f_-^2+(\partial_3+m_3)f_-^3-(\partial_0-m_0)f_-^0]
\end{pmatrix} \\
\begin{pmatrix}
(\partial_0+m_0)E_+^3+(\partial_1-m_1)B_+^2-(\partial_2-m_2)B_+^1+ \\
\quad -(\partial_3+m_3)[(\partial_1-m_1)f_+^1+(\partial_2-m_2)f_+^2+(\partial_3-m_3)f_+^3-(\partial_0+m_0)f_+^0] \\
(\partial_0-m_0)E_-^3+(\partial_1+m_1)B_-^2-(\partial_2+m_2)B_-^1+ \\
\quad -(\partial_3-m_3)[(\partial_1+m_1)f_-^1+(\partial_2+m_2)f_-^2+(\partial_3+m_3)f_-^3-(\partial_0-m_0)f_-^0]
\end{pmatrix} \\
\begin{pmatrix}
(\partial_1-m_1)E_+^1+(\partial_1-m_1)E_+^2-(\partial_1-m_1)E_+^3+ \\
\quad +(\partial_0-m_0)[(\partial_1-m_1)f_+^1+(\partial_2-m_2)f_+^2+(\partial_3-m_3)f_+^3-(\partial_0+m_0)f_+^0] \\
(\partial_1+m_1)E_-^1+(\partial_1+m_1)E_-^2+(\partial_1+m_1)E_-^3+ \\
\quad +(\partial_0+m_0)[(\partial_1+m_1)f_-^1+(\partial_2+m_2)f_-^2+(\partial_3+m_3)f_-^3-(\partial_0-m_0)f_-^0]
\end{pmatrix}
\end{pmatrix}
\tag{20}
$$

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$$\left(\Box-|m|^{2}\right)\mathbf{f}=\left(\Box-|m|^{2}\right)\begin{pmatrix}\mathbf{f}_{+}\\ \mathbf{f}_{-}\end{pmatrix}=\left(\Box-|m|^{2}\right)\begin{pmatrix}\begin{pmatrix}f_{+}^{1}\\ f_{+}^{2}\\ f_{+}^{3}\\ f_{+}^{0}\end{pmatrix}\\ \begin{pmatrix}f_{-}^{1}\\ f_{-}^{2}\\ f_{-}^{3}\\ f_{-}^{0}\end{pmatrix}\end{pmatrix}=\begin{pmatrix}\begin{pmatrix}J_{+}^{1}\\ J_{+}^{2}\\ J_{+}^{3}\\ J_{+}^{0}\end{pmatrix}\\ \begin{pmatrix}J_{-}^{1}\\ J_{-}^{2}\\ J_{-}^{3}\\ J_{-}^{0}\end{pmatrix}\end{pmatrix}=\begin{pmatrix}\mathbf{J}_{+}\\ \mathbf{J}_{-}\end{pmatrix}=\mathrm{J}=$$

$$=\begin{pmatrix}\begin{pmatrix}(\partial_0+m_0)E_{+}^{1}+(\partial_2-m_2)B_{+}^{3}-(\partial_3-m_3)B_{+}^{2}+\\ \qquad-(\partial_1+m_1)[(\partial_1-m_1)f_{+}^{1}+(\partial_2-m_2)f_{+}^{2}+(\partial_3-m_3)f_{+}^{3}-(\partial_0+m_0)f_{+}^{0}]\\ (\partial_0+m_0)E_{+}^{2}+(\partial_1-m_1)B_{+}^{3}-(\partial_3-m_3)B_{+}^{1}+\\ \qquad-(\partial_2+m_2)[(\partial_1+m_1)f_{+}^{1}+(\partial_2-m_2)f_{+}^{2}+(\partial_3-m_3)f_{+}^{3}-(\partial_0+m_0)f_{+}^{0}]\\ (\partial_0+m_0)E_{+}^{3}+(\partial_1-m_1)B_{+}^{2}-(\partial_2-m_2)B_{+}^{1}+\\ \qquad-(\partial_3+m_3)[(\partial_1-m_1)f_{+}^{1}+(\partial_2-m_2)f_{+}^{2}+(\partial_3-m_3)f_{+}^{3}-(\partial_0+m_0)f_{+}^{0}]\\ (\partial_1-m_1)E_{+}^{1}+(\partial_1-m_1)E_{+}^{2}+(\partial_1-m_1)E_{+}^{3}+\\ \qquad+(\partial_0-m_0)[(\partial_1-m_1)f_{-}^{1}+(\partial_2+m_2)f_{+}^{2}+(\partial_3-m_3)f_{+}^{3}-(\partial_0+m_0)f_{+}^{0}]\end{pmatrix}\\ \begin{pmatrix}(\partial_0-m_0)E_{-}^{1}+(\partial_2+m_2)B_{-}^{3}-(\partial_3+m_3)B_{-}^{2}+\\ \qquad-(\partial_1-m_1)[(\partial_1+m_1)f_{-}^{1}+(\partial_2+m_2)f_{-}^{2}+(\partial_3+m_3)f_{-}^{3}-(\partial_0-m_0)f_{-}^{0}]\\ (\partial_0-m_0)E_{-}^{2}+(\partial_1+m_1)B_{-}^{3}-(\partial_3+m_3)B_{-}^{1}+\\ \qquad-(\partial_2-m_2)[(\partial_1+m_1)f_{-}^{1}+(\partial_2+m_2)f_{-}^{2}+(\partial_3+m_3)f_{-}^{3}-(\partial_0-m_0)f_{-}^{0}]\\ (\partial_0-m_0)E_{-}^{3}+(\partial_1+m_1)B_{-}^{2}-(\partial_2+m_2)B_{-}^{1}+\\ \qquad-(\partial_3-m_3)[(\partial_1+m_1)f_{-}^{1}+(\partial_2+m_2)f_{-}^{2}+(\partial_3-m_3)f_{-}^{3}-(\partial_0-m_0)f_{-}^{0}]\\ (\partial_1+m_1)E_{-}^{1}+(\partial_1+m_1)E_{-}^{2}+(\partial_1+m_1)E_{-}^{3}+\\ \qquad+(\partial_0+m_0)[(\partial_1+m_1)f_{-}^{1}+(\partial_2+m_2)f_{-}^{2}+(\partial_3+m_3)f_{-}^{3}-(\partial_0-m_0)f_{-}^{0}]\end{pmatrix}\end{pmatrix}\qquad(21)$$

Rearranging:

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$$=\begin{pmatrix}\begin{pmatrix}\left(D_0^+E_+^1+D_2^-B_+^3-D_3^-B_+^2\right)-D_1^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^+E_+^2+D_1^-B_+^3-D_3^-B_+^1\right)-D_2^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^+E_+^3+D_1^-B_+^2-D_2^-B_+^1\right)-D_3^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_1^-E_+^1+D_2^-E_+^2+D_3^-E_+^3\right)+D_0^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\\ \begin{pmatrix}\left(D_0^-E_-^1+D_2^+B_-^3-D_3^+B_-^2\right)-D_1^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^-E_-^2+D_1^+B_-^3-D_3^+B_-^1\right)-D_2^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^-E_-^3+D_1^+B_-^2-D_2^+B_-^1\right)-D_3^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_1^+E_-^1+D_2^+E_-^2+D_3^+E_-^3\right)+D_0^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\end{pmatrix}=\begin{pmatrix}\begin{pmatrix}J_+^1\\J_+^2\\J_+^3\\J_+^0\end{pmatrix}\\ \begin{pmatrix}J_-^1\\J_-^2\\J_-^3\\J_-^0\end{pmatrix}\end{pmatrix}=\begin{pmatrix}\mathbf{J}_+\\ \mathbf{J}_-\end{pmatrix}=\mathbf{J}=\begin{pmatrix}\begin{pmatrix}-\mathbf{J}_+\\ \rho_+\end{pmatrix}\\ \begin{pmatrix}-\mathbf{J}_-\\ \rho_-\end{pmatrix}\end{pmatrix}=$$

$$=\begin{pmatrix}\begin{pmatrix}\left(D_0^+E_+^1+D_2^-B_+^3-D_3^-B_+^2\right)-D_1^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^+E_+^2+D_1^-B_+^3-D_3^-B_+^1\right)-D_2^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^+E_+^3+D_1^-B_+^2-D_2^-B_+^1\right)-D_3^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_1^-E_+^1+D_2^-E_+^2+D_3^-E_+^3\right)+D_0^+\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\\ \begin{pmatrix}\left(D_0^-E_-^1+D_2^+B_-^3-D_3^+B_-^2\right)-D_1^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^-E_-^2+D_1^+B_-^3-D_3^+B_-^1\right)-D_2^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_0^-E_-^3+D_1^+B_-^2-D_2^+B_-^1\right)-D_3^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\\ \left(D_1^+E_-^1+D_2^+E_-^2+D_3^+E_-^3\right)+D_0^-\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\end{pmatrix}\quad(22)$$

$$=\begin{pmatrix}\begin{pmatrix}(\partial_0+m_0)\vec{\mathbf{E}}_+-\left(\vec{\nabla}-\vec{\mathbf{m}}\right)\times\vec{B}_+-\overset{m}{\vec{\nabla}}\Box\left(\overset{m}{\nabla}_{\Updownarrow}\Box\ \mathbf{f}^*\right)\\ \left(\vec{\nabla}-\vec{\mathbf{m}}\right)\Box\vec{\mathbf{E}}_++(\partial_0-m_0)\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\\ \begin{pmatrix}(\partial_0-m_0)\vec{\mathbf{E}}_--\left(\vec{\nabla}+\vec{m}\right)\times\vec{B}_--\overset{m}{\vec{\nabla}}\Box\left(\overset{m}{\nabla}_{\Updownarrow}\Box\ \mathbf{f}^*\right)\\ \left(\vec{\nabla}+\vec{\mathbf{m}}\right)\Box\vec{\mathbf{E}}_-+(\partial_0+m_0)\left(\overset{m}{\nabla}_{\Updownarrow}\Box\mathbf{f}^*\right)\end{pmatrix}\end{pmatrix}\quad(23)$$

Since the mass constituents/components of anti-fermions are the negative of their corresponding fermion, these may be simply written:

$$\begin{pmatrix}\mathbf{J}\\ \rho\end{pmatrix}=\begin{pmatrix}(\partial_0+m_0)\vec{\mathbf{E}}-\left(\vec{\nabla}-\vec{\mathbf{m}}\right)\times\vec{\mathbf{B}}-\overset{m}{\nabla}\cdot\left(\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ \left(\vec{\nabla}-\vec{\mathbf{m}}\right)\cdot\vec{\mathbf{E}}+(\partial_0-m_0)\left(\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\end{pmatrix}\tag{24}$$

Equations (15) may be subtracted to yield:

$$\mathbf{0}=\begin{pmatrix}-D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1\\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2\\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3\\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow}\end{pmatrix}\begin{pmatrix}B_{\Updownarrow}^1+E^1\\ B_{\Updownarrow}^2+E^2\\ B_{\Updownarrow}^3+E^3\\ -\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\end{pmatrix}-\begin{pmatrix}D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1\\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2\\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3\\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow}\end{pmatrix}\begin{pmatrix}B_{\Updownarrow}^1-E^1\\ B_{\Updownarrow}^2-E^2\\ B_{\Updownarrow}^3-E^3\\ \overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\end{pmatrix}\tag{25}$$

$$=\begin{pmatrix}-D_0B_{\Updownarrow}^1-D_0E^1+D_3^{\Leftrightarrow}B_{\Updownarrow}^2+D_3^{\Leftrightarrow}E^2-D_2^{\Leftrightarrow}B_{\Updownarrow}^3-D_2^{\Leftrightarrow}E^3-D_1\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ -D_3^{\Leftrightarrow}B_{\Updownarrow}^1-D_3^{\Leftrightarrow}E^1-D_0B_{\Updownarrow}^2-D_0E^2+D_1^{\Leftrightarrow}B_{\Updownarrow}^3+D_1^{\Leftrightarrow}E^3-D_2\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ D_2^{\Leftrightarrow}B_{\Updownarrow}^1+D_2^{\Leftrightarrow}E^1-D_1^{\Leftrightarrow}B_{\Updownarrow}^2-D_1^{\Leftrightarrow}E^2-D_0B_{\Updownarrow}^3-D_0E^3-D_3\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ -D_1^{\Updownarrow}B_{\Updownarrow}^1-D_1^{\Updownarrow}E^1-D_2^{\Updownarrow}B_{\Updownarrow}^2-D_2^{\Updownarrow}E^2-D_3^{\Updownarrow}B_{\Updownarrow}^3-D_3^{\Updownarrow}E^3+D_0^{\Updownarrow}\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\end{pmatrix}+$$

$$-\begin{pmatrix}D_0B_{\Updownarrow}^1-D_0E^1+D_3^{\Leftrightarrow}B_{\Updownarrow}^2-D_3^{\Leftrightarrow}E^2-D_2^{\Leftrightarrow}B_{\Updownarrow}^3+D_2^{\Leftrightarrow}E^3+D_1\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ -D_3^{\Leftrightarrow}B_{\Updownarrow}^1+D_3^{\Leftrightarrow}E^1+D_0B_{\Updownarrow}^2-D_0E^2+D_1^{\Leftrightarrow}B_{\Updownarrow}^3-D_1^{\Leftrightarrow}E^3+D_2\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ D_2^{\Leftrightarrow}B_{\Updownarrow}^1-D_2^{\Leftrightarrow}E^1-D_1^{\Leftrightarrow}B_{\Updownarrow}^2+D_1^{\Leftrightarrow}E^2+D_0B_{\Updownarrow}^3-D_0E^3+D_3\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\\ D_1^{\Updownarrow}B_{\Updownarrow}^1-D_1^{\Updownarrow}E^1+D_2^{\Updownarrow}B_{\Updownarrow}^2-D_2^{\Updownarrow}E^2+D_3^{\Updownarrow}B_{\Updownarrow}^3-D_3^{\Updownarrow}E^3-D_0^{\Updownarrow}\left(-\overset{m}{\nabla}_{\Updownarrow}\cdot\mathbf{f}^*\right)\end{pmatrix}\tag{26}$$

$$=\begin{pmatrix}-2D_0B_{\Updownarrow}^1+2D_3^{\Leftrightarrow}E^2-2D_2^{\Leftrightarrow}E^3\\ -2D_3^{\Leftrightarrow}E^1-2D_0B_{\Updownarrow}^2+2D_1^{\Leftrightarrow}E^3\\ 2D_2^{\Leftrightarrow}E^1-2D_1^{\Leftrightarrow}E^2-2D_0B_{\Updownarrow}^3\\ -2D_1^{\Updownarrow}B_{\Updownarrow}^1-2D_2^{\Updownarrow}B_{\Updownarrow}^2-2D_3^{\Updownarrow}B_{\Updownarrow}^3\end{pmatrix}=-2\begin{pmatrix}D_0B_{\Updownarrow}^1+(D_2^{\Leftrightarrow}E^3-D_3^{\Leftrightarrow}E^2)\\ D_0B_{\Updownarrow}^2+(D_3^{\Leftrightarrow}E^1-D_1^{\Leftrightarrow}E^3)\\ D_0B_{\Updownarrow}^3+(D_1^{\Leftrightarrow}E^2-D_2^{\Leftrightarrow}E^1)\\ D_1^{\Updownarrow}B_{\Updownarrow}^1+D_2^{\Updownarrow}B_{\Updownarrow}^2+D_3^{\Updownarrow}B_{\Updownarrow}^3\end{pmatrix}\tag{27}$$

$$=\begin{pmatrix}\begin{pmatrix}(\partial_0+m_0)B_-^1+[(\partial_2-m_2)E_-^3-(\partial_3-m_3)E_-^2]\\ (\partial_0-m_0)B_+^1-[(\partial_2+m_2)E_+^3-(\partial_3+m_3)E_+^2]\end{pmatrix}\\ \begin{pmatrix}(\partial_0+m_0)B_-^2+[(\partial_3-m_3)E_-^1-(\partial_1-m_1)E_-^3]\\ (\partial_0-m_0)B_+^2-[(\partial_3+m_3)E_+^1-(\partial_1+m_1)E_+^3]\end{pmatrix}\\ \begin{pmatrix}(\partial_0+m_0)B_-^3+[(\partial_1-m_1)E_-^2-(\partial_2-m_2)E_-^1]\\ (\partial_0-m_0)B_+^3-[(\partial_1+m_1)E_+^2-(\partial_2+m_2)E_+^1]\end{pmatrix}\\ \begin{pmatrix}(\partial_1-m_1)B_-^1+[(\partial_2-m_2)B_-^2+(\partial_3-m_3)B_-^3\\ (\partial_1+m_1)B_+^1+[(\partial_2+m_2)B_+^2+(\partial_3+m_3)B_+^3\end{pmatrix}\end{pmatrix}\tag{28}$$

$$
= \begin{pmatrix} \begin{pmatrix} (\partial_0 - m_0)B_+^1 + [(\partial_2 + m_2)E_+^3 - (\partial_3 + m_3)E_+^2] \\ (\partial_0 - m_0)B_+^2 + [(\partial_3 + m_3)E_+^1 - (\partial_1 + m_1)E_+^3] \\ (\partial_0 - m_0)B_+^3 + [(\partial_1 + m_1)E_+^2 - (\partial_2 + m_2)E_+^1] \\ (\partial_1 + m_1)B_+^1 + [(\partial_2 + m_2)B_+^2 + (\partial_3 + m_3)B_+^3) \end{pmatrix} \\ \begin{pmatrix} (\partial_0 + m_0)B_-^1 + [(\partial_2 - m_2)E_-^3 - (\partial_3 - m_3)E_-^2] \\ (\partial_0 + m_0)B_-^2 + [(\partial_3 - m_3)E_-^1 - (\partial_1 - m_1)E_-^3] \\ (\partial_0 + m_0)B_-^3 + [(\partial_1 - m_1)E_-^2 - (\partial_2 - m_2)E_-^1] \\ (\partial_1 - m_1)B_-^1 + [(\partial_2 - m_2)B_-^2 + (\partial_3 - m_3)B_-^3) \end{pmatrix} \end{pmatrix} \tag{29}
$$

$$
= \begin{pmatrix} \begin{pmatrix} (\partial_0 - m_0)\vec{\mathbf{B}}_+ + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}}_+ \\ (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}_+ \end{pmatrix} \\ \begin{pmatrix} (\partial_0 + m_0)\vec{\mathbf{B}}_- + (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{E}}_- \\ (\vec{\nabla} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}_- \end{pmatrix} \end{pmatrix} \tag{30}
$$

Again, since the mass constituents/components of anti-fermions are the negative of their corresponding fermion, these may be simply written:

$$
\mathbf{0} = \begin{pmatrix} (\partial_0 - m_0)\vec{\mathbf{B}} + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}} \\ (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}} \end{pmatrix} \tag{31}
$$

Thus, using the boundary (gauge) condition $\left(\overset{m}{\nabla}_{\Updownarrow} \cdot \mathbf{f}^{*}\right) = 0$ these mass-generalized Maxwell's equations may be simply written:

$0 = (\partial_0 - m_0)\vec{\mathbf{B}} + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}};$	$0 = (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}$	; Homgeneous
$\vec{\mathbf{J}} = (\partial_0 + m_0)\vec{\mathbf{E}} - (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}};$	$\rho = (\vec{\nabla} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}}$	; Inhomgeneous

(32)

Modestly christened the **Maxwell-Cassano equations** of an electromagnetic-nuclear field.

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